The steps to an uncertainty budget

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Metrology for Earth Observation and Climate
http://www.emceoc.org

EMRP
European Metrology Research Programme
Programme of EURAMET

The EMRP is jointly funded by the EMRP participating countries within EURAMET and the European Union

EURAMET European Association of National Metrology Institutes

Uncertainty

- Where to start?
- What to do?
- How to be consistent?
- Make it easy.

At the end of this module, you should understand

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- Uncertainty analysis is a multi-step process
  - Understanding the problem
  - Determining the formal relationships
  - Propagating the uncertainties

- How to develop an uncertainty budget
  - 8 steps to an uncertainty budget

- There is no single right way
  - Mathematical / modelling
  - Experimental
  - Combination

8 steps to an uncertainty budget

- Understanding the problem

  Step 1: Describing the Traceability Chain

  Step 2: Writing down the calculation equations

  Step 3: Considering the sources of uncertainty

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  Step 4: Creating the measurement equation

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UNCERTAINTY ANALYSIS IS A MULTI-STEP PROCESS

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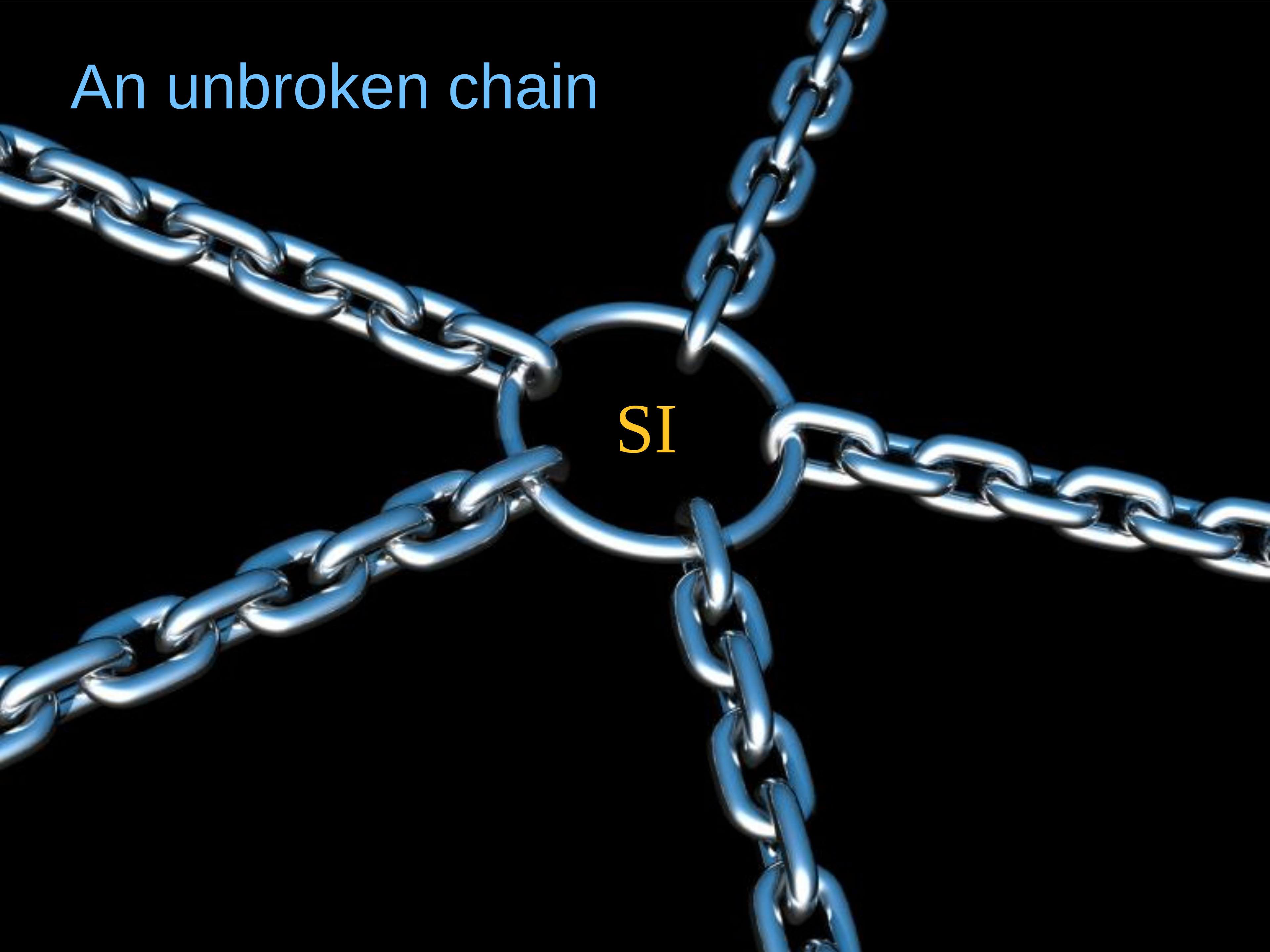
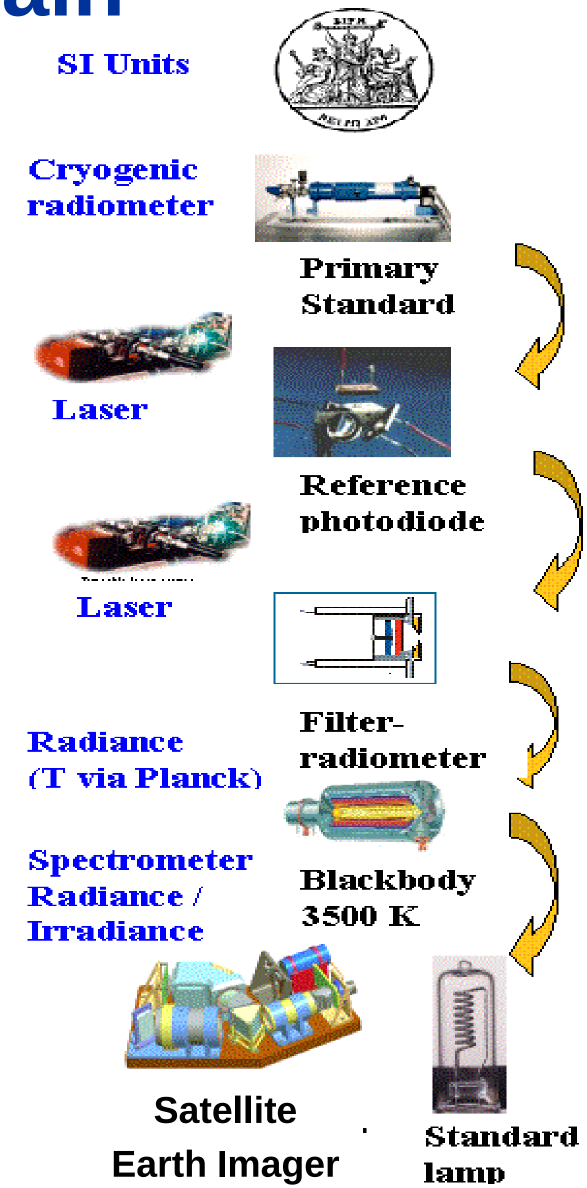
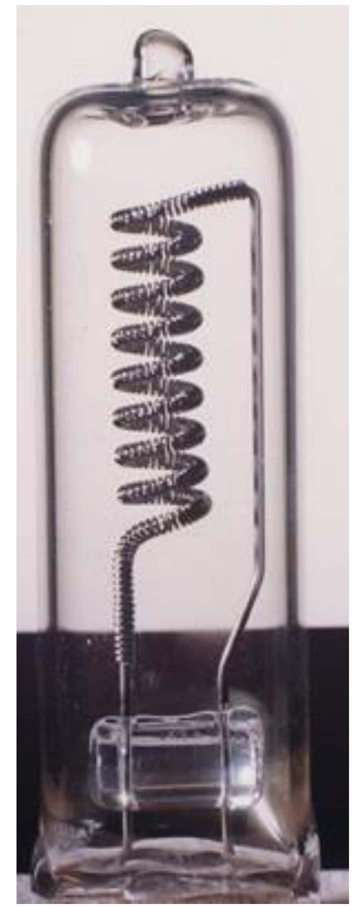
An unbroken chain

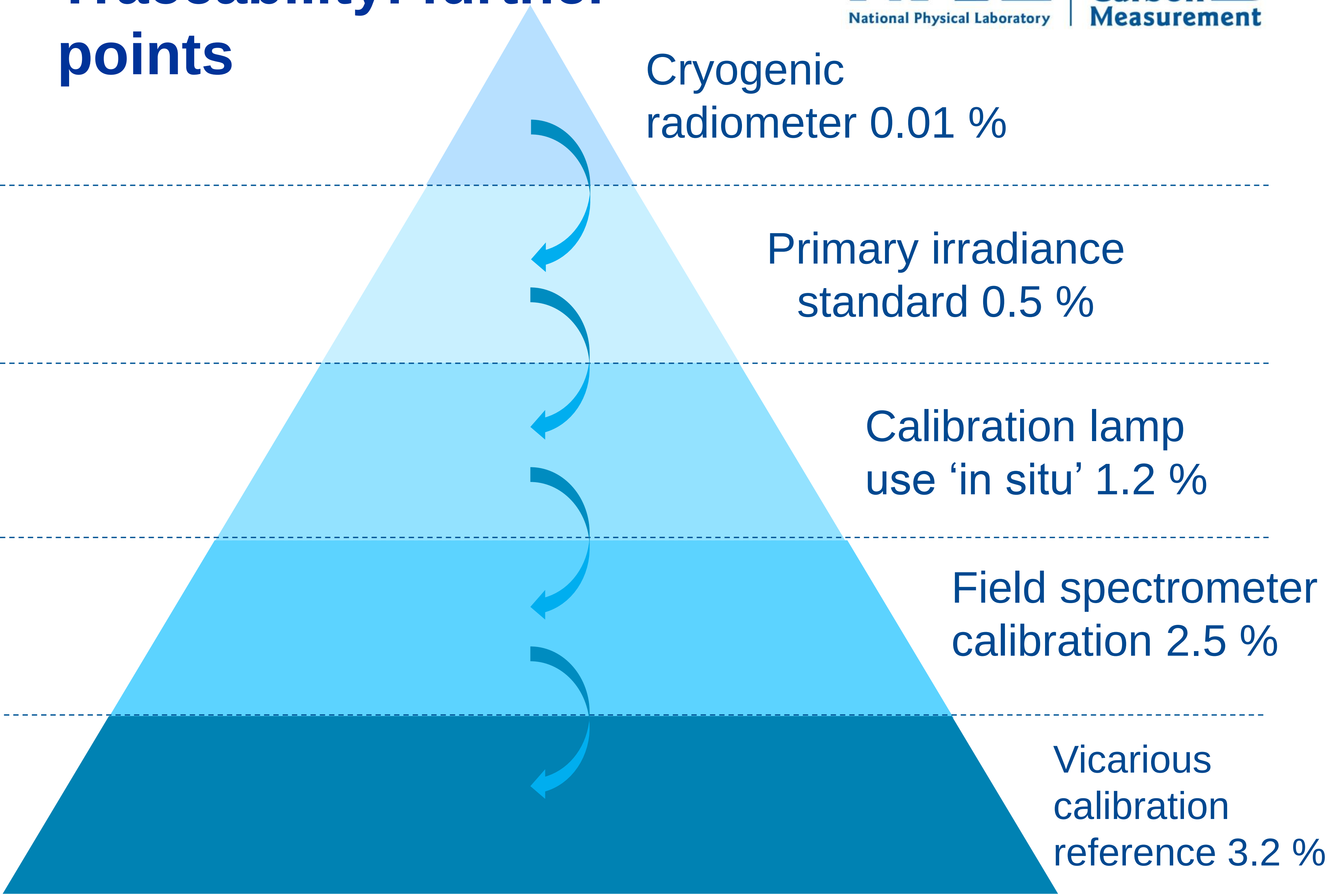


Describing the Traceability Chain

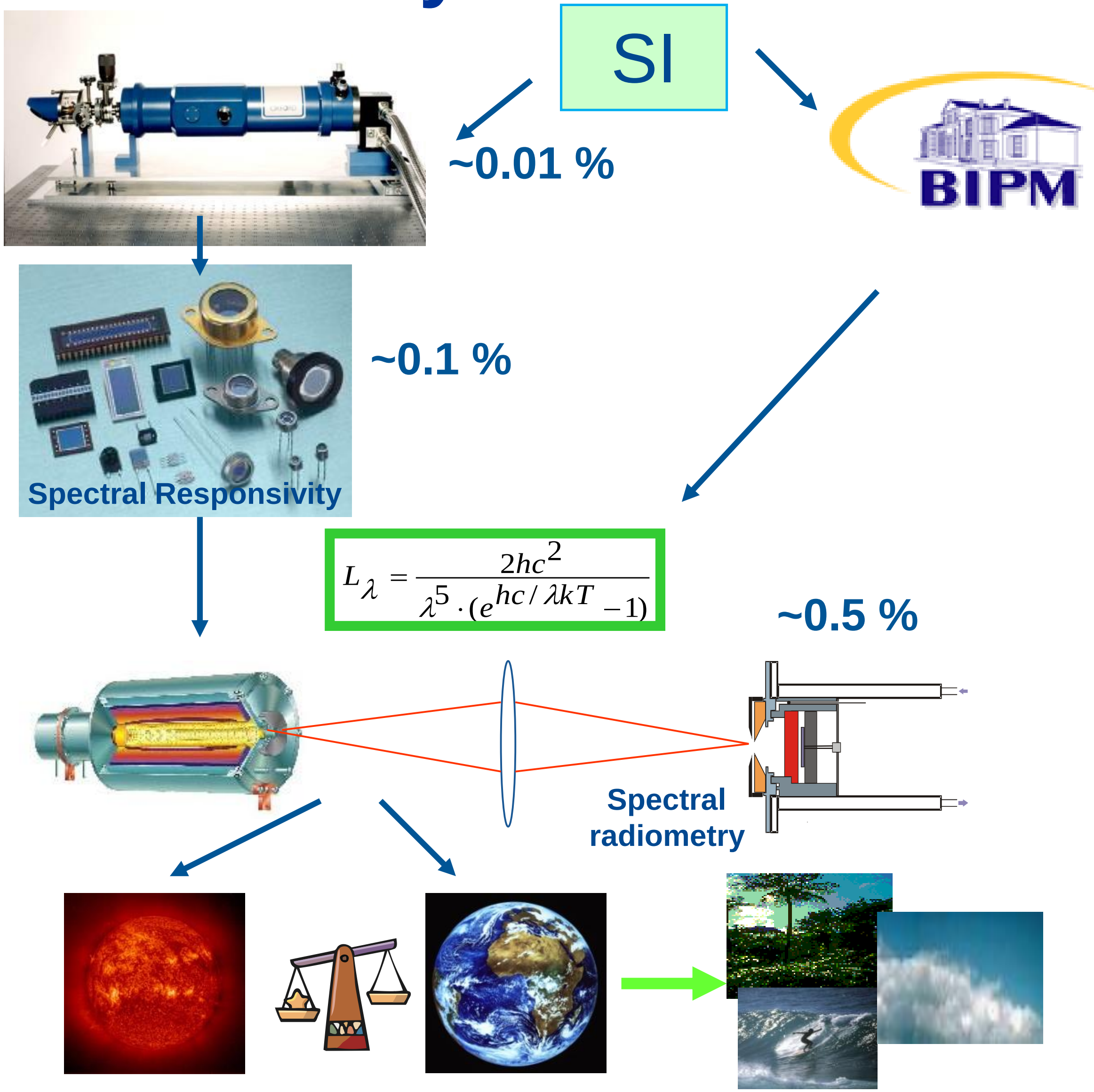


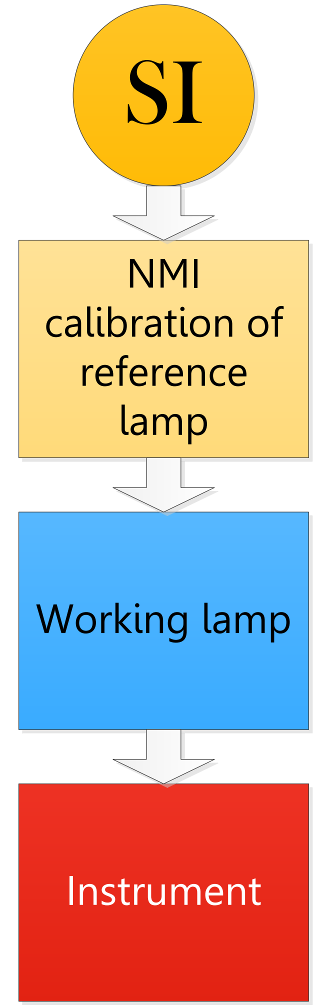


Traceability: further points

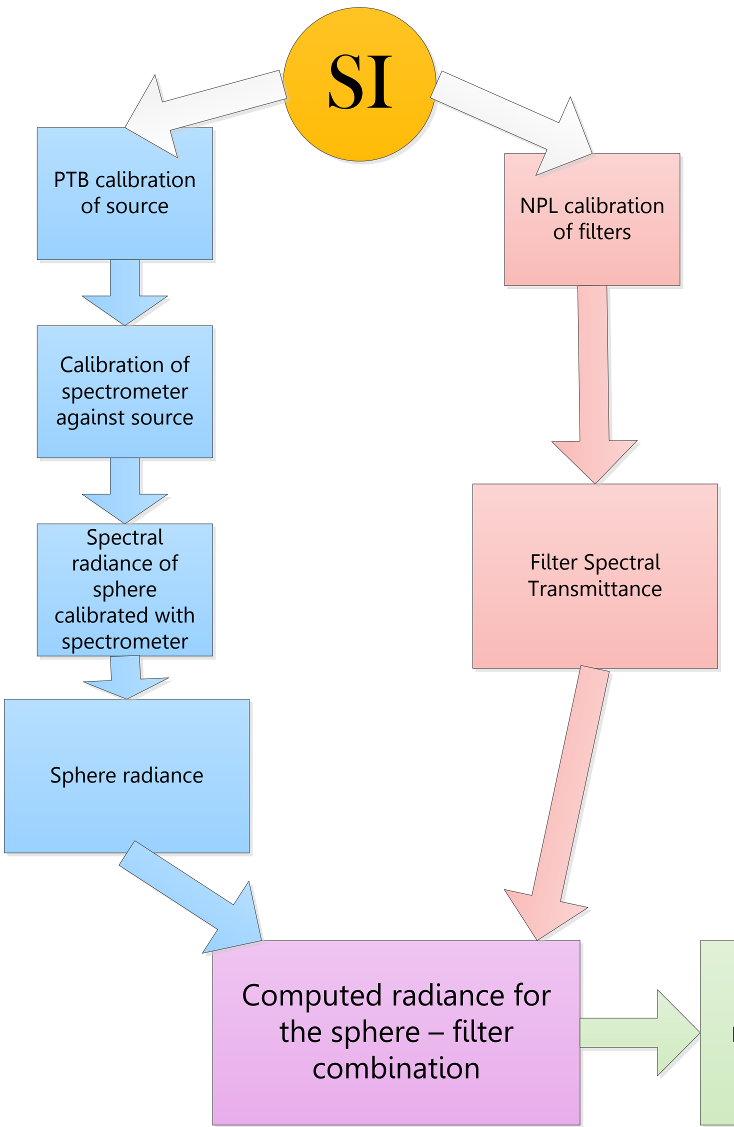


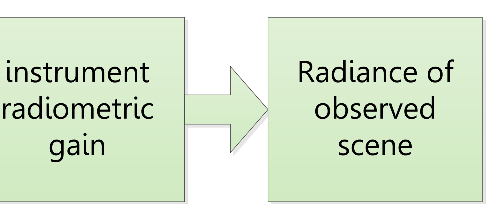
Describing the Traceability Chain





Describing the Traceability Chain





Understanding the problem

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Writing down the calculation equations

$$L_s = \frac{E_{\mathrm{FEL}} \beta_{0-45}}{\pi}$$

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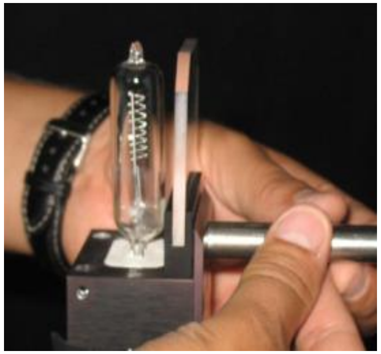
Considering the sources of uncertainty

Considering the sources of uncertainty

Lamp additional effects

- Ageing
- Alignment
- Current stability



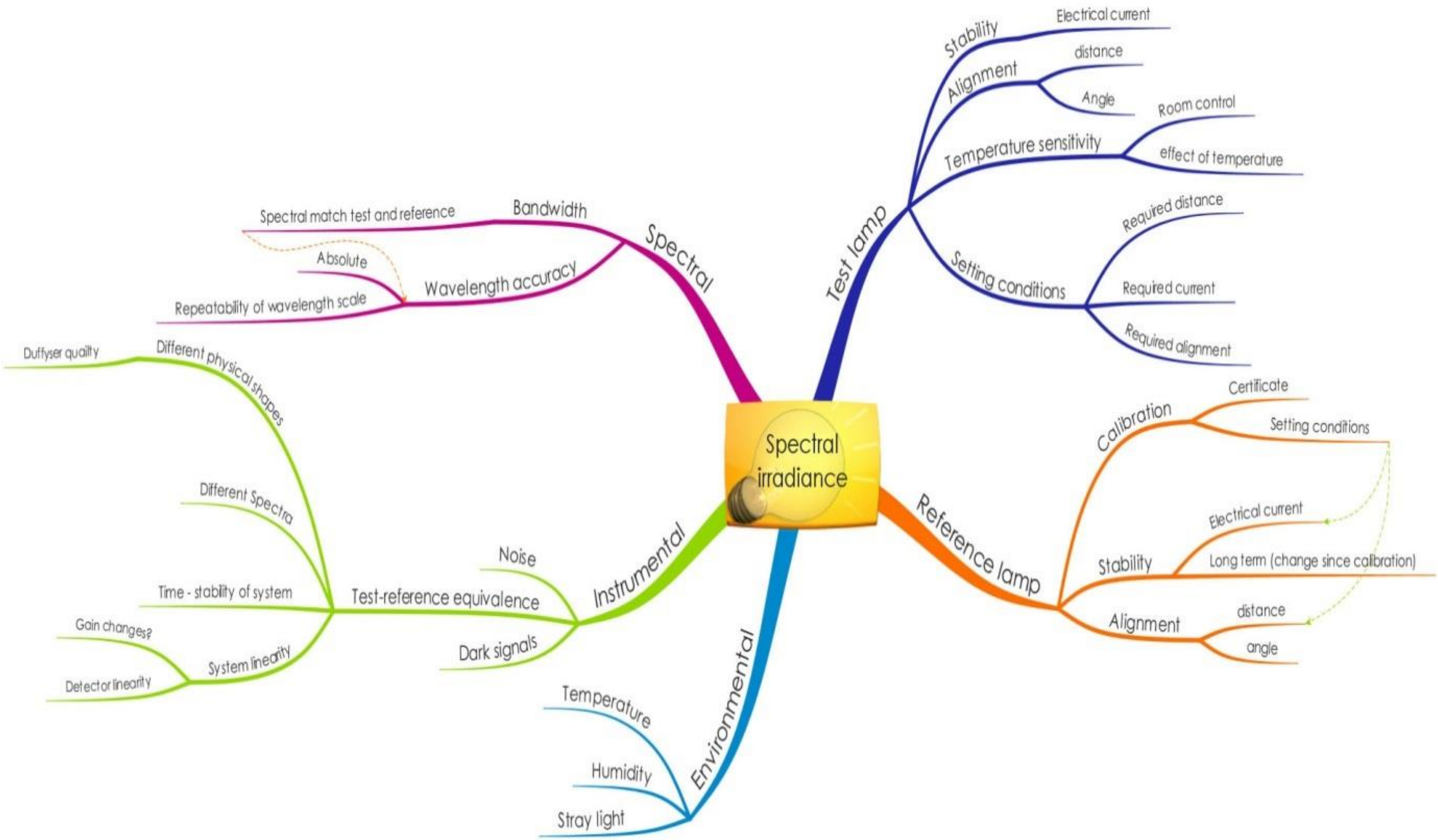
Distance accuracy

Diffuser additional effects

- Ageing
- Uniformity

Random noise

Considering the sources of uncertainty



Determining the formal relationships

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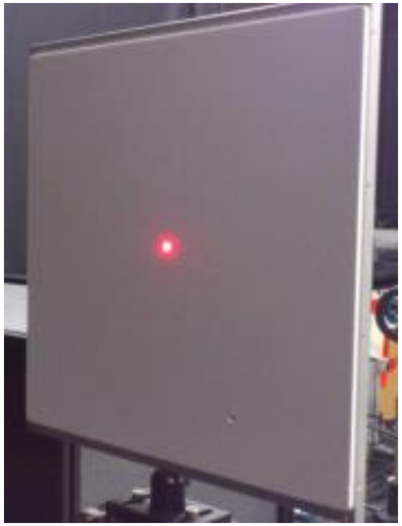
Creating the measurement equation



$$L_s = \frac{E_{FEL}\beta_{0-45}}{\pi}$$

$$L_s = \frac{E_{FEL}\beta_{0\text{-}45}}{\pi}\frac{d_{cal}^2}{d_{use}^2} K_{lamp\_stab} K_{align} K_{current} K_{diff\_stab} K_{unif}$$

$$L_s = \frac{E_{FEL}\beta_{0-45}}{\pi}\frac{d_{cal}^2}{d_{use}^2}$$

Determining the formal relationships

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Determining the sensitivity coefficients

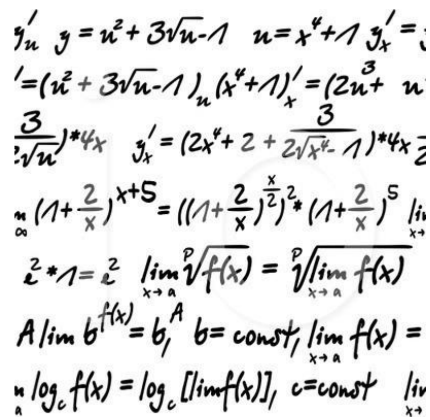
$$u_c^2(y) = \sum_{i=1}^{n}\left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j}u(x_i, x_j)$$

- There is no single right way
  - **Mathematical / modelling**
  - **Experimental**
  - **Combination**

Determining the sensitivity coefficients

- Do an experiment

- Analytical expression



- Model it

Determining the formal relationships

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Assigning uncertainties

Uncertainty component	Associated uncertainty		(relative)	Uncertainty associated with radiance due to this
	absolute	relative	Sensitivity coefficient	
Lamp irradiance (calibration)		0.30%	1	0.30%
Diffuser reflectance factor (calibration)		0.30%	1	0.30%
Lamp-diffuser distance (same as calibration distance for lamp)?	1 mm in 500 mm	0.20%	2	0.40%
Stability of lamp (short term)		0.10%	1	0.10%
Stability of lamp (drift/ageing)		0.10%	1	0.10%
Alignment of lamp				0.05%
Current stability of lamp (at 350 nm)	3 mA			0.29%
Diffuser stability (ageing)		0.10%	1	0.10%
Uniformity of diffuser		0.50%	1	0.50%

$$L_s = \frac{E_{FEL}\beta_{0\text{-}45}}{\pi}\frac{d_{cal}^2}{d_{use}^2} K_{lamp\_stab} K_{align} K_{current} K_{diff\_stab} K_{unif}$$

Propagating the uncertainties

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Combining and propagating uncertainties

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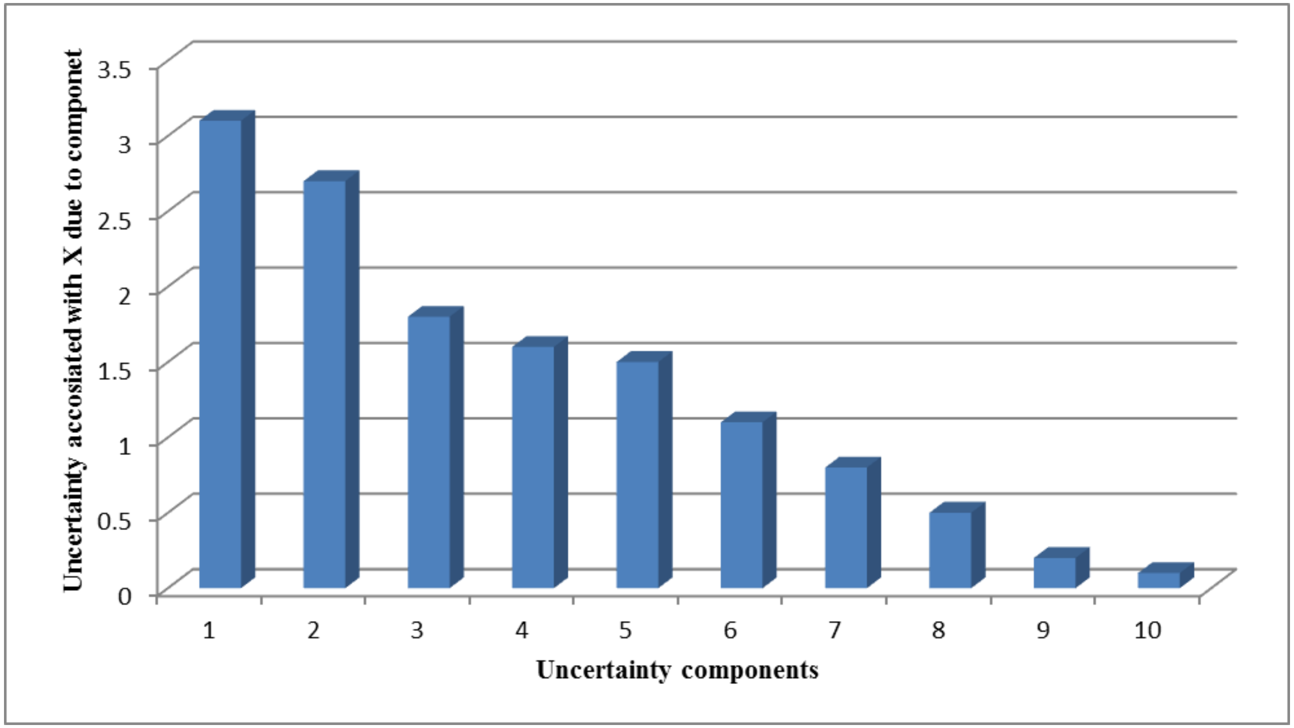
$$u_c^2(y) = \sum_{i=1}^{n}\left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j}u(x_i,x_j)$$

Has a sensitivity coefficient
Adding in quadrature (% or units)

Averages reduce by $1/\sqrt{n}$

This term is to do with correlation

Combining and propagating uncertainties



$$u(X) = 5.20\%$$

Propagating the uncertainties

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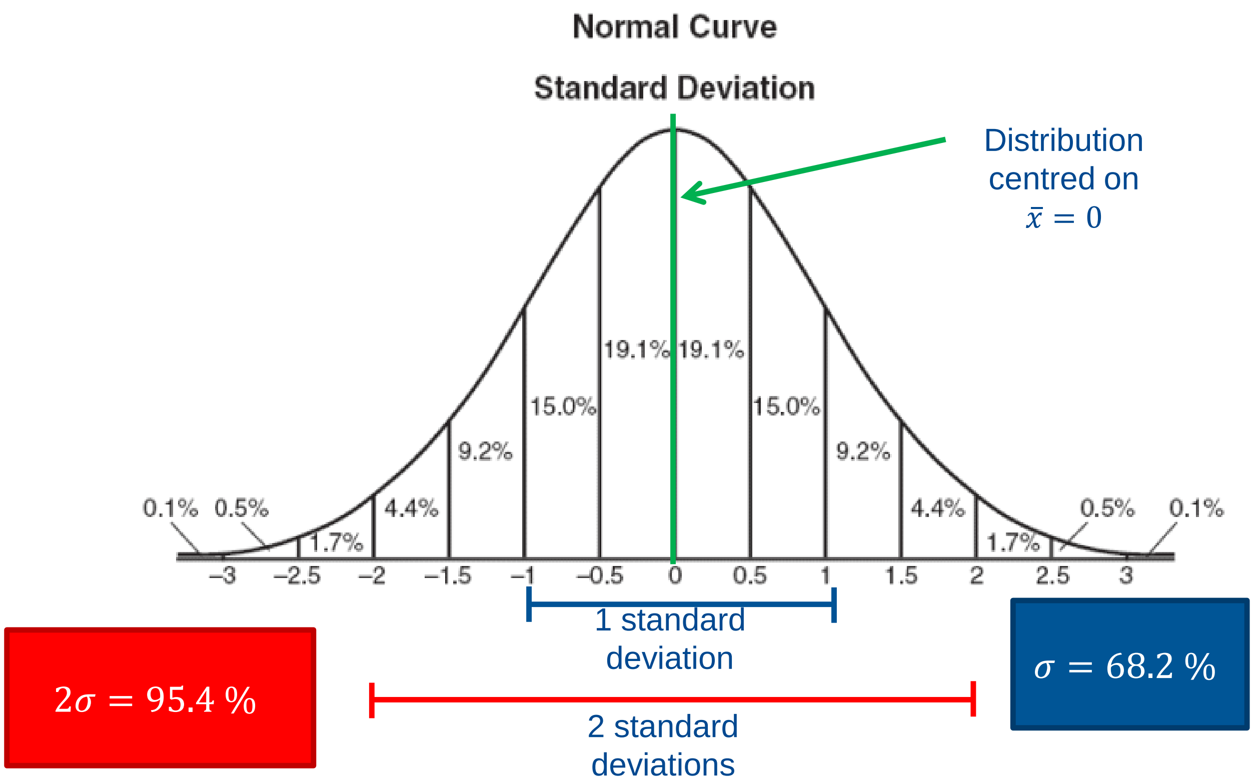
Expanding uncertainties



If the distribution is not Gaussian, then a different coverage factor is needed.

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