The steps to an uncertainty budget

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Uncertainty

• Where to start?

• What to do?

• How to be consistent?

• Make it easy.
At the end of this module, you should understand

- Uncertainty analysis is a multi-step process
  - Understanding the problem
  - Determining the formal relationships
  - Propagating the uncertainties

- How to develop an uncertainty budget
  - 8 steps to an uncertainty budget

- There is no single right way
  - Mathematical / modelling
  - Experimental
  - Combination
8 steps to an uncertainty budget

- Understanding the problem
  Step 1: Describing the Traceability Chain
  Step 2: Writing down the calculation equations
  Step 3: Considering the sources of uncertainty

- Determining the formal relationships
  Step 4: Creating the measurement equation
  Step 5: Determining the sensitivity coefficients
  Step 6: Assigning uncertainties

- Propagating the uncertainties
  Step 7: Combining and propagating uncertainties
  Step 8: Expanding uncertainties
UNCERTAINTY ANALYSIS IS A MULTI-STEP PROCESS
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Understanding the problem

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An unbroken chain
Describing the Traceability Chain

SI Units

Cryogenic radiometer

Laser

Primary Standard

Reference photodiode

Radiance (T via Planck)

Spectrometer Radiance / Irradiance

Filter-radiometer

Blackbody 3500 K

Satellite Earth Imager

Standard lamp
Traceability: further points

- **Cryogenic radiometer** 0.01%
- **Primary irradiance standard** 0.5%
- **Calibration lamp use ‘in situ’** 1.2%
- **Field spectrometer calibration** 2.5%
- **Vicarious calibration reference** 3.2%
Describing the Traceability Chain

\[ L_\lambda = \frac{2hc^2}{\lambda^5 \cdot (e^{hc/\lambda kT} - 1)} \]

\[ \sim 0.01\% \]

\[ \sim 0.1\% \]

\[ \sim 0.5\% \]
Describing the Traceability Chain

- Sphere radiance
  - Calibrated spectral radiance
    - Spectral radiance of sphere calibrated with spectrometer
    - Calibration of spectrometer against source
    - PTB calibration of source

- Filter Spectral Transmittance
  - NPL calibration of filters

- Computed radiance for the sphere – filter combination
  - Radiance of observed scene
  - Instrument radiometric gain

Calibration Home Base at DLR

Images of calibration equipment and calibration scenes.
Understanding the problem

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Writing down the calculation equations

\[ L_s = \frac{E_{\text{FEL}} \beta_{0-45}}{\pi} \]
Understanding the problem

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Considering the sources of uncertainty
Considering the sources of uncertainty

Lamp additional effects
- Ageing
- Alignment
- Current stability

Distance accuracy

Diffuser additional effects
- Ageing
- Uniformity

Random noise
Considering the sources of uncertainty
Determining the formal relationships

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Creating the measurement equation

\[ L_s = \frac{E_{FEL} \beta_{0-45}}{\pi} \]

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Determining the formal relationships

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Determining the sensitivity coefficients

\[ u_c^2 (y) = \sum_{i=1}^{n} \left( \frac{\partial f}{\partial x_i} \right)^2 u^2 (x_i) + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j) \]

- There is no single right way
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Determining the sensitivity coefficients

- Do an experiment
- Analytical expression
- Model it
Determining the formal relationships

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## Assigning uncertainties

<table>
<thead>
<tr>
<th>Uncertainty component</th>
<th>Associated uncertainty</th>
<th>(relative)</th>
<th>Sensitivity coefficient</th>
<th>Uncertainty associated with radiance due to this</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lamp irradiance (calibration)</td>
<td>0.30%</td>
<td>1</td>
<td>1</td>
<td>0.30%</td>
</tr>
<tr>
<td>Diffuser reflectance factor (calibration)</td>
<td>0.30%</td>
<td>1</td>
<td>1</td>
<td>0.30%</td>
</tr>
<tr>
<td>Lamp-diffuser distance (same as calibration distance for lamp)?</td>
<td>1 mm in 500 mm</td>
<td>0.20%</td>
<td>2</td>
<td>0.40%</td>
</tr>
<tr>
<td>Stability of lamp (short term)</td>
<td>0.10%</td>
<td>1</td>
<td>1</td>
<td>0.10%</td>
</tr>
<tr>
<td>Stability of lamp (drift/ageing)</td>
<td>0.10%</td>
<td>1</td>
<td>1</td>
<td>0.10%</td>
</tr>
<tr>
<td>Alignment of lamp</td>
<td></td>
<td></td>
<td>0.05%</td>
<td></td>
</tr>
<tr>
<td>Current stability of lamp (at 350 nm)</td>
<td>3 mA</td>
<td></td>
<td></td>
<td>0.29%</td>
</tr>
<tr>
<td>Diffuser stability (ageing)</td>
<td>0.10%</td>
<td>1</td>
<td>1</td>
<td>0.10%</td>
</tr>
<tr>
<td>Uniformity of diffuser</td>
<td>0.50%</td>
<td>1</td>
<td></td>
<td>0.50%</td>
</tr>
</tbody>
</table>

\[
L_s = \frac{E_{FEL}}{\pi \beta_{0-45}} \frac{d^2_{cal}}{d^2_{use}} K_{\text{lamp stab}} K_{\text{align}} K_{\text{current}} K_{\text{diff stab}} K_{\text{unif}}
\]
Propagating the uncertainties

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- Has a sensitivity coefficient
- Adding in quadrature (% or units)
- Averages reduce by \(1/\sqrt{n}\)

This term is to do with correlation
Combining and propagating uncertainties

\[ u(X) = 5.20\% \]
Propagating the uncertainties

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Expanding uncertainties

If the distribution is not Gaussian, then a different coverage factor is needed.
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