Measurement Equation

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Metrology for Earth Observation and Climate
http://www.emceoc.org

At the end of this module, you should understand

- The difference between calculation and measurement equations
  - Why the measurement equation has more components
  - Where do they come from

- How to develop a measurement equation
  - How to evaluate additional components in the measurement equation
  - When to stop adding them

- How to determine the sensitivity coefficients
  - Analytically
  - Modelling
  - Lab experiment

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  - Lab experiment

Lamp-diffuser

Integrating sphere

Laboratory based

Lamp – diffuser method

RADIANCE CALIBRATION 1

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Lamp-diffuser

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  - How to evaluate additional components in the measurement equation
  - When to stop adding them

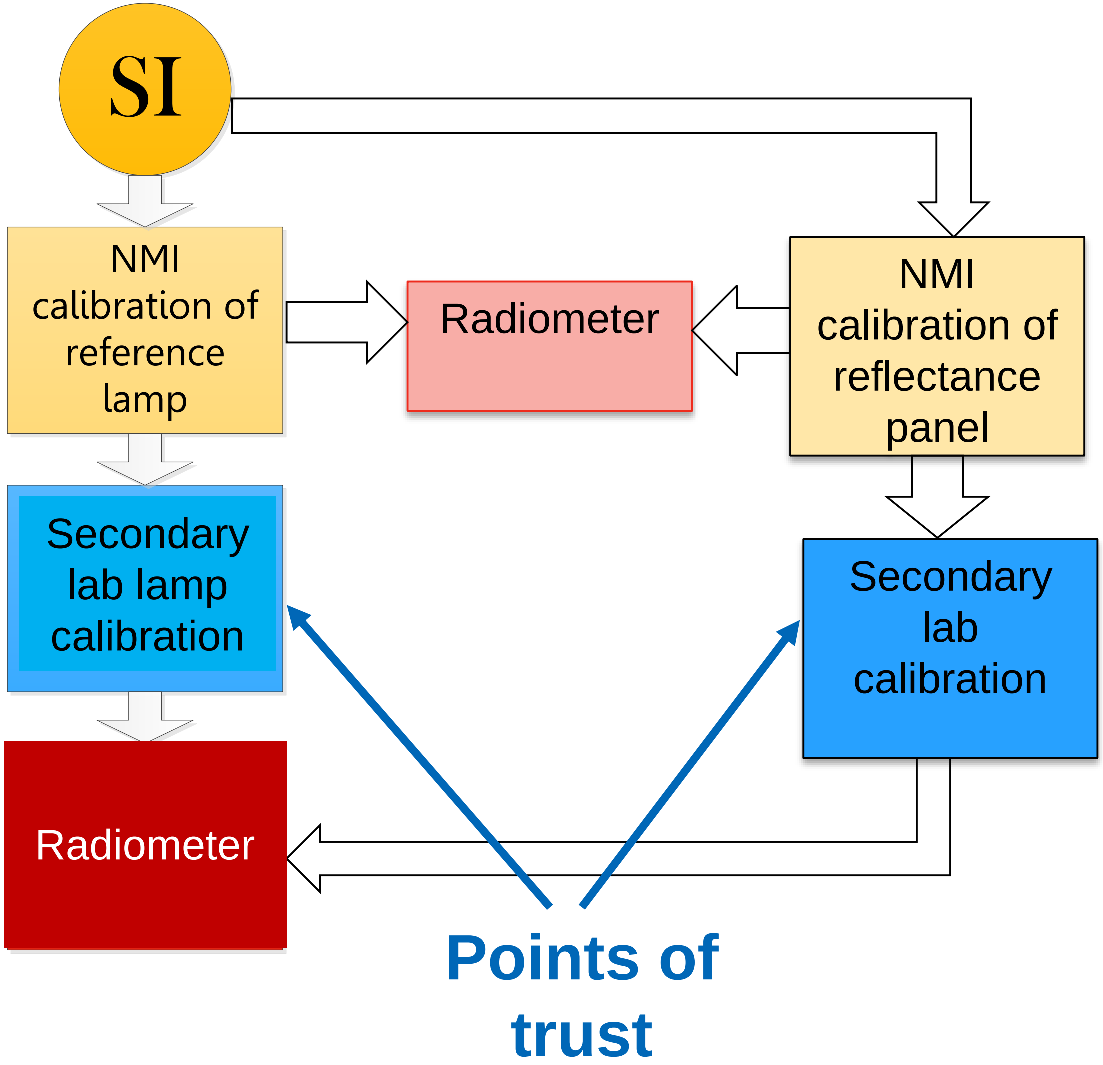
- How to determine the sensitivity coefficients
  - Analytically
  - Modelling
  - Lab experiment

Steps to an uncertainty budget

1. Traceability Chain
2. Calculation Equation
3. Sources of Uncertainty
4. Measurement Equation
5. Sensitivity Coefficients
6. Assigning Uncertainties
7. Combining your uncertainties
8. Expanding your uncertainties

Symbol	Uncertainty component	Size of effect	Correction applied?	Residual uncertainty	Divisor	Sensitivity coefficient	Uncertainty associated with final value due to effect
Combined standard uncertainty							
Expanded uncertainty							

Step 1: Traceability chain



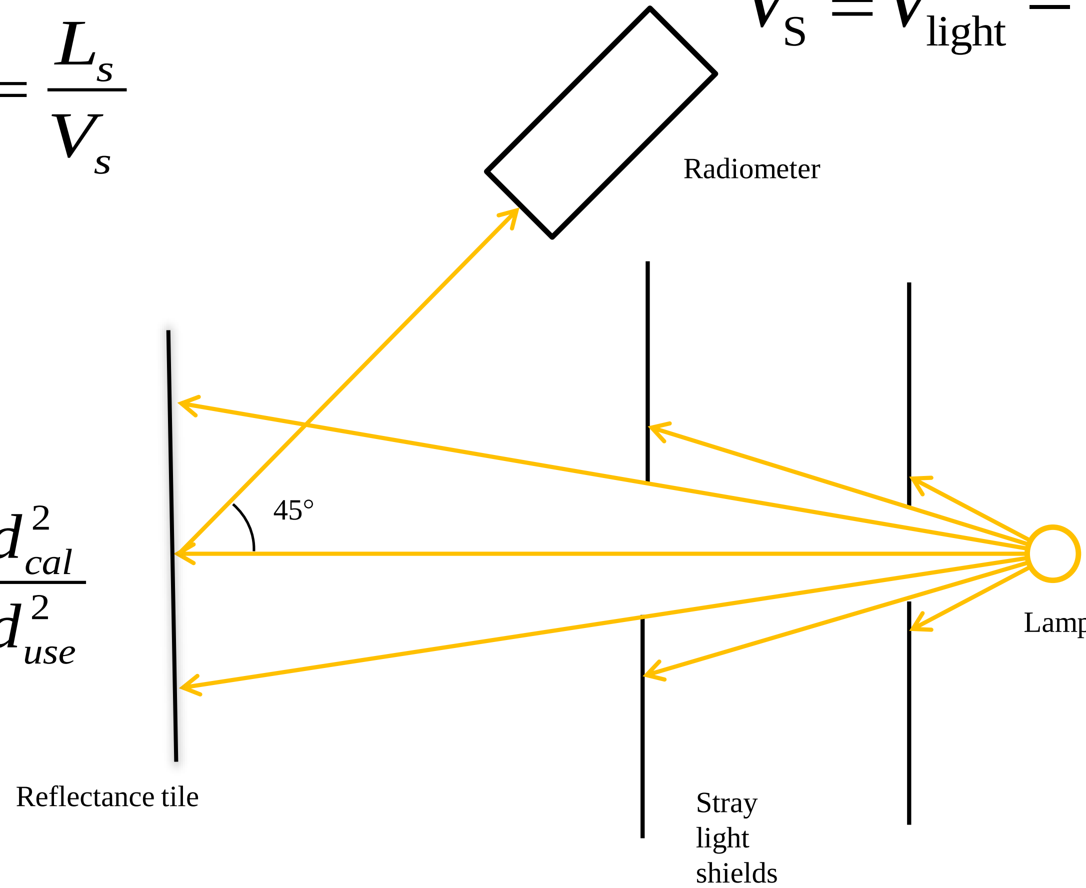
Step 2: Calculation equations

Absolute calibration coefficient

$$A_L = \frac{L_s}{V_s}$$

$$V_S = V_{\text{light}} - V_{\text{dark}}$$

$$L_s = \frac{E_{FEL}\,\beta_{0:45}}{\pi}\,\frac{d_{cal}^2}{d_{use}^2}$$



Step 3: Sources of uncertainty

Calibration certificate

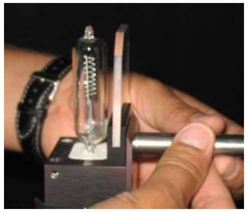
Lamp additional effects

- Ageing
- Alignment
- Current stability

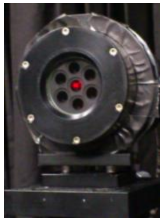
Calibration certificate

Diffuser additional effects

- Ageing
- Uniformity



Distance accuracy



Random noise

Instrument additional effects

- Stability (drift)
- Room stray light

Step 4: Measurement equations

$$\boxed{L_s = \frac{E_{FEL}\beta_{0:45}}{\pi} \frac{d_{cal}^2}{d_{use}^2}}$$

$$L_s = \frac{E_{FEL}\beta_{0:45}}{\pi} \frac{d_{cal}^2}{d_{use}^2} K_{lamp\_stab} K_{align} K_{current} K_{diff\_stab} K_{unif}$$

$$\boxed{V_S = V_{light} - V_{dark}}$$

$$V_S = V_{light} K_{light\_stab} + K_{stray} - V_{dark} K_{dark\_stab}$$

Steps to an uncertainty budget

1. Traceability Chain
2. Calculation Equation
3. Sources of Uncertainty
4. Measurement Equation
5. **Sensitivity Coefficients**
6. **Assigning Uncertainties**
7. Combining your uncertainties
8. Expanding your uncertainties

Sensitivity coefficients (1)

- **By differentiation**

Easy to derive for ***calculation equation*** components of this particular example

Component x_i	Sensitivity coefficient $c_i = \frac{\partial f}{\partial x_i}$	Relative radiance uncertainty due to … $c_i \mu(x_i)$
Lamp irradiance E_{FEL}	L_s / E_{FEL}	$1 \cdot u(E_{FEL}) / E_{FEL}$
Radiance factor $\beta_{0:45}$	$L_s / \beta_{0:45}$	$1 \cdot u(\beta_{0:45}) / \beta_{0:45}$
Distance d_{use}^2	$-2L_s / d_{use}$	$-2 \cdot u(d_{use}) / d_{use}$

$$\left(\frac{u(L)}{L}\right)^2 = \left(\frac{u(E_{FEL})}{E_{FEL}}\right)^2 + \left(\frac{u(\beta_{0:45})}{\beta_{0:45}}\right)^2 + (-2)^2 \left(\frac{u(d_{use})}{d_{use}}\right)^2$$

Uncertainties of calculation equation components (1)

From calibration certificates

Component x_i	Sensitivity coefficient $c_i = \frac{\partial f}{\partial x_i}$	Relative radiance uncertainty due to … $c_i u(x_i)$
Lamp irradiance E_{FEL}	L_s / E_{FEL}	$1 \cdot u(E_{FEL}) / E_{FEL}$
Radiance factor $\beta_{0:45}$	$L_s / \beta_{0:45}$	$1 \cdot u(\beta_{0:45}) / \beta_{0:45}$
Distance d_{use}^2	$-2L_s / d_{use}$	$-2 \cdot u(d_{use}) / d_{use}$

$$\left(\frac{u(L)}{L}\right)^2 = \left(\frac{u(E_{FEL})}{E_{FEL}}\right)^2 + \left(\frac{u(\beta_{0:45})}{\beta_{0:45}}\right)^2 + (-2)^2\left(\frac{u(d_{use})}{d_{use}}\right)^2$$

Certificate uncertainties (1)

$$L_s = \frac{E_{FEL}\beta_{0:45}}{\pi}\frac{d_{cal}^2}{d_{use}^2} K_{lamp\_stab} K_{align} K_{current} K_{diff\_stab} K_{unif}$$

From calibration certificates

Wave-length	Absolute Spectral Irradiance	Uncertainty
nm	mW m^{-2} nm^{-1}	%
545	4.37	1.5
550	4.54	1.5
555	4.72	1.5
560	4.89	1.4
565	5.07	1.6
570	5.24	1.5
575	5.42	1.5
580	5.60	1.4
585	5.78	1.5
590	5.96	1.4
595	6.14	1.4
600	6.31	1.3
605	6.48	1.3
610	6.65	1.3
615	6.83	1.3
620	7.00	1.2
625	7.18	1.2
630	7.35	1.2
635	7.52	1.3
640	7.68	1.3

Remember calibration certificates almost always quote uncertainties at k = 2 !

Certificate uncertainties (2)

$$L_s = \frac{E_{FEL}\beta_{0:45}}{\pi}\frac{d_{cal}^2}{d_{use}^2} K_{lamp\_stab} K_{align} K_{current} K_{diff\_stab} K_{unif}$$

Radiometric quantities integrated over passband function of radiometer being calibrated, allowing for its spectral response function; may require interpolation

Wavelength [nm]	Spectral Irradiance [W/(cm² nm)]
250	1.831E-08
260	3.198E-08
270	5.264E-08
280	8.263E-08
290	1.247E-07
300	1.813E-07
310	2.555E-07
320	3.501E-07
330	4.678E-07
340	6.121E-07
350	7.854E-07
360	9.868E-07
370	1.220E-06
380	1.485E-06
390	1.791E-06
400	2.119E-06
450	4.281E-06
500	7.078E-06
555	1.051E-05
600	1.323E-05
654.6	1.624E-05
700	1.829E-05
800	2.117E-05
900	2.212E-05
1050	2.104E-05
1150	1.944E-05

390
400
450
500
555
600
654.6
700
800
900
1050
1150

See course book Chapter 7 for more details about integration and interpolation

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Certificate uncertainties (3)

$$L_{\mathrm{s}} = \frac{E_{\mathrm{FEL}}\beta_{0:45}}{\pi}\frac{d_{\mathrm{cal}}^{2}}{d_{\mathrm{use}}^{2}} K_{\mathrm{lamp\_stab}} K_{\mathrm{align}} K_{\mathrm{current}} K_{\mathrm{diff\_stab}} K_{\mathrm{unif}}$$

$$\rho \neq \beta_{0:45}$$

Certificate may not give required quantity (modelling may be needed to obtain desired quantity): read it carefully!

Measurement equation: allowing for additional effects

$$L_s = \frac{E_{\mathrm{FEL}}\rho}{\pi}\frac{d_{\mathrm{cal}}^2}{d_{\mathrm{use}}^2}\, all\_Ks$$

including

$$K_{\mathrm{lamp\_interp}} K_{\rho \Rightarrow \beta_{0:45}}$$

$$K_{\rho \Rightarrow \beta_{0:45}} \neq 1$$

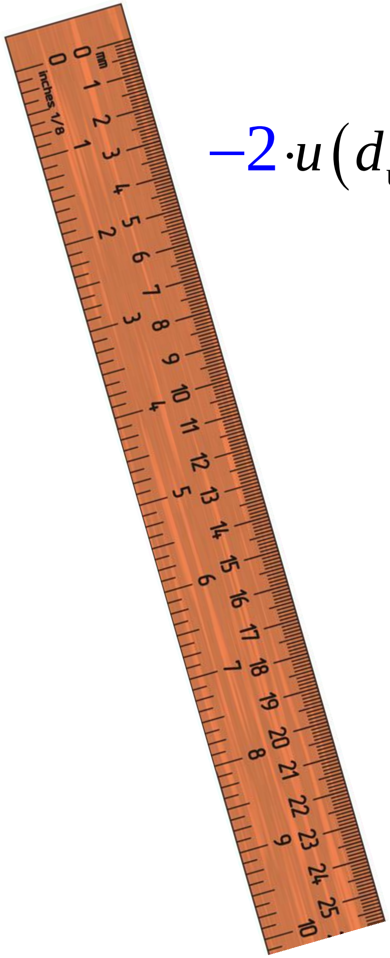
Uncertainties of calculation equation components (2)

e.g. from resolution of distance measuring instrument

Component x_i	Sensitivity coefficient $c_i = \dfrac{\partial f}{\partial x_i}$	Relative radiance uncertainty due to … $c_i u(x_i)$
Lamp irradiance E_{FEL}	L_s / E_{FEL}	$1 \cdot u(E_{\mathrm{FEL}}) / E_{\mathrm{FEL}}$
Radiance factor $\beta_{0:45}$	$L_s / \beta_{0:45}$	$1 \cdot u(\beta_{0:45}) / \beta_{0:45}$
Distance d_{use}^2	$-2 L_s / d_{\mathrm{use}}$	$-2 \cdot u(d_{\mathrm{use}}) / d_{\mathrm{use}}$

$$\left(\frac{u(L)}{L}\right)^2 = \left(\frac{u(E_{\mathrm{FEL}})}{E_{\mathrm{FEL}}}\right)^2 + \left(\frac{u(\beta_{0:45})}{\beta_{0:45}}\right)^2 + (-2)^2 \left(\frac{u(d_{\mathrm{use}})}{d_{\mathrm{use}}}\right)^2$$

Rectangular uncertainty distributions



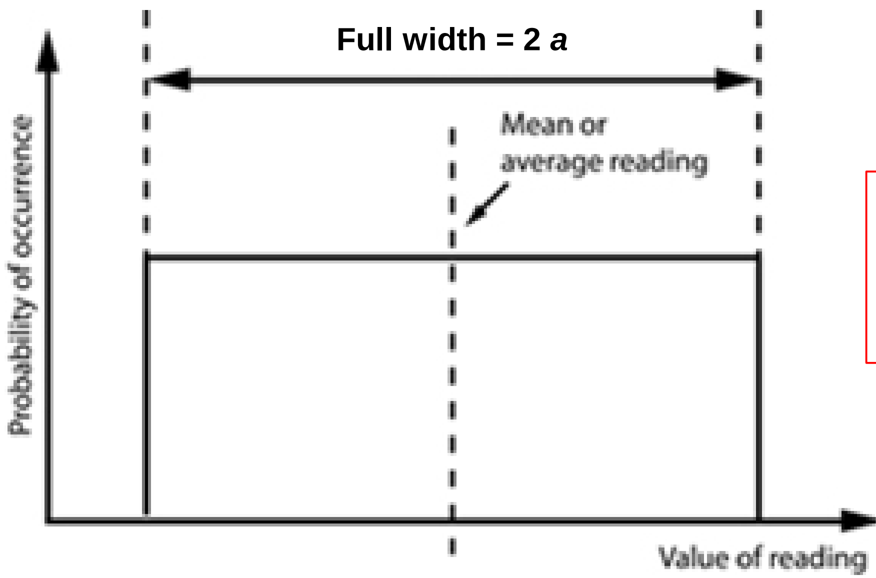
$$-2\cdot u\left(d_{\text{use}}\right)/d_{\text{use}}$$

Resolution of distance measuring instrument = 0.1 mm

Measurement distance = 500.0 mm

Uncertainty associated with distance measurement =
(0.05 / 500) / √3 = 0.006 %

Uncertainty in irradiance from distance measurement =
2 × 0.006 % = 0.012 %



$$\boxed{\frac{a}{\sqrt{3}}}$$

Uncertainties of calculation equation components (3)

$$\left(\frac{u(L)}{L}\right)^2=\left(\frac{u(E_{\mathrm{FEL}})}{E_{\mathrm{FEL}}}\right)^2+\left(\frac{u(\beta_{0:45})}{\beta_{0:45}}\right)^2+(-2)^2\left(\frac{u(d_{\mathrm{use}})}{d_{\mathrm{use}}}\right)^2$$

Symbol	Uncertainty component	Size of effect	Correction applied?	Residual uncertainty	Divisor	Sensitivity coefficient	Uncertainty associated with final value due to effect
$u(E_{\mathrm{FEL}})$	Ref. lamp irradiance	1.5 %	N	1.5 %	2	1	0.75 %
$u(\beta_{0:45})$	Tile radiance factor	2.0 %	N	2.0 %	2	1	1.00 %
$u(d_{\mathrm{use}})$	Lamp distance (500 mm)	0.05 mm	N	0.01 %	$\sqrt{3}$	2	0.012 %
$u(d_{\mathrm{use}})$							
$u(d_{\mathrm{use}})$							
$u(d_{\mathrm{use}})$							
$u(d_{\mathrm{use}})$							
$u(d_{\mathrm{use}})$							
$u(d_{\mathrm{use}})$							
$u(d_{\mathrm{use}})$							
$u(d_{\mathrm{use}})$							
Combined standard uncertainty							
Expanded uncertainty							

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Uncertainty of additional effects (1)

$$L_s = \frac{E_{\mathrm{FEL}}\beta_{0:45}}{\pi}\frac{d_{\mathrm{cal}}^2}{d_{\mathrm{use}}^2} K_{\mathrm{lamp\_stab}} K_{\mathrm{align}} K_{\mathrm{current}} K_{\mathrm{diff\_stab}} K_{\mathrm{unif}}$$

Statistical analysis of repeated measurements

$$V_S = V_{\mathrm{light}} K_{\mathrm{light\_stab}} + K_{\mathrm{stray}} - V_{\mathrm{dark}} K_{\mathrm{dark\_stab}}$$

Repeat measurements with realignment of the lamp

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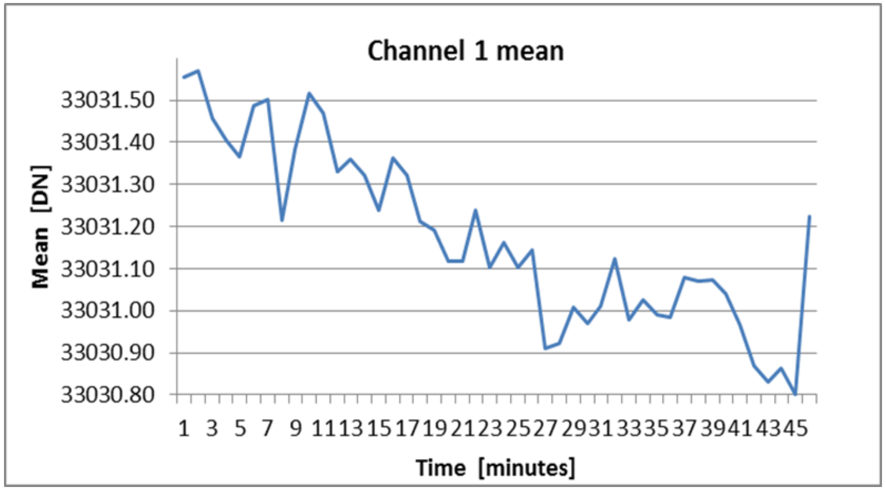
Uncertainty of additional effects (2)

- Negligible instrument drift in controlled lab environment



0.7 DN change during
45 minute constant run

$$V_S = V_{\mathrm{light}} \cancel{K_{\mathrm{light\_stab}}} + K_{\mathrm{stray}} - V_{\mathrm{dark}} \cancel{K_{\mathrm{dark\_stab}}}$$

Uncertainty of additional effects (3)

$$L_s = \frac{E_{FEL} \beta_{0:45}}{\pi} \frac{d_{cal}^2}{d_{use}^2} K_{lamp\_stab} K_{align} K_{current} K_{diff\_stab} K_{unif}$$

Historical calibration records; data from other researchers

$$V_S = V_{light} K_{light\_stab} + K_{stray} - V_{dark} K_{dark\_stab}$$

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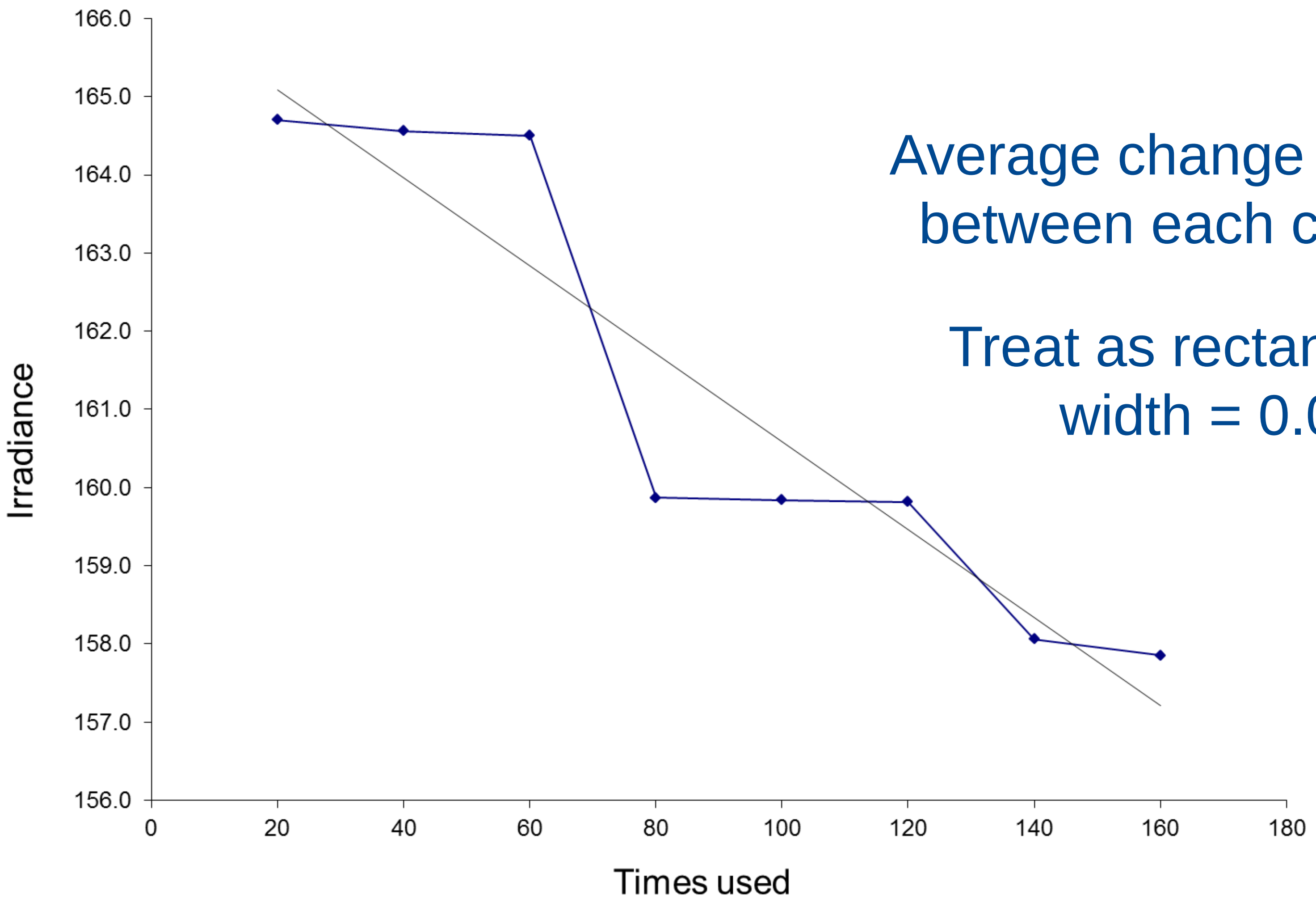
Uncertainty of additional effects (4)



Average change = 0.083 % between each calibration

Treat as rectangular, half width = 0.083 %

Uncertainty of additional effects (5)

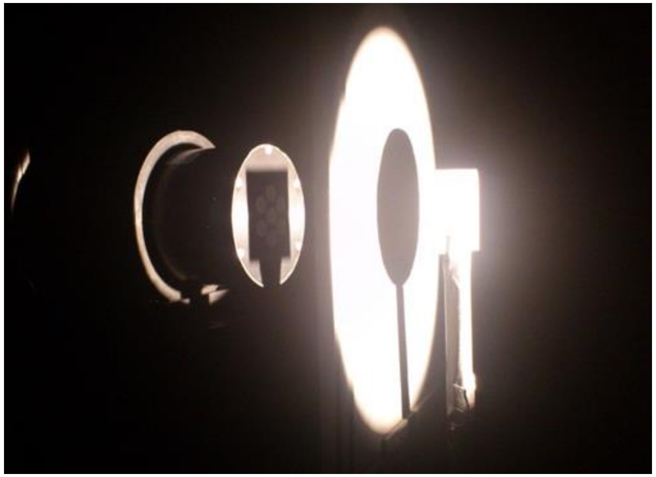
$$L_s = \frac{E_{\mathrm{FEL}}\beta_{0:45}}{\pi}\frac{d_{\mathrm{cal}}^2}{d_{\mathrm{use}}^2}K_{\mathrm{lamp\_stab}}K_{\mathrm{align}}K_{\mathrm{current}}K_{\mathrm{diff\_stab}}K_{\mathrm{unif}}$$

By measurement

$$V_S = V_{\mathrm{light}}K_{\mathrm{light\_stab}} + K_{\mathrm{stray}} - V_{\mathrm{dark}}K_{\mathrm{dark\_stab}}$$

Uncertainty of additional effects (6)

- Room stray light negligible



Difference between detector dark reading and measurement with detector FOV obscured smaller than standard deviation of individual dark runs

$$V_S = V_{\text{light}} K_{\text{light\_stab}} + \cancel{K_{\text{stray}}} - V_{\text{dark}} K_{\text{dark\_stab}}$$

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Uncertainty of additional effects (7)

$$L_s = \frac{E_{\mathrm{FEL}}\beta_{0:45}}{\pi}\frac{d_{\mathrm{cal}}^2}{d_{\mathrm{use}}^2} K_{\mathrm{lamp\_stab}} K_{\mathrm{align}} K_{\mathrm{current}} K_{\mathrm{diff\_stab}} K_{\mathrm{unif}}$$

By modelling (or by measurement)

$$V_S = V_{\mathrm{light}} K_{\mathrm{light\_stab}} + K_{\mathrm{stray}} - V_{\mathrm{dark}} K_{\mathrm{dark\_stab}}$$

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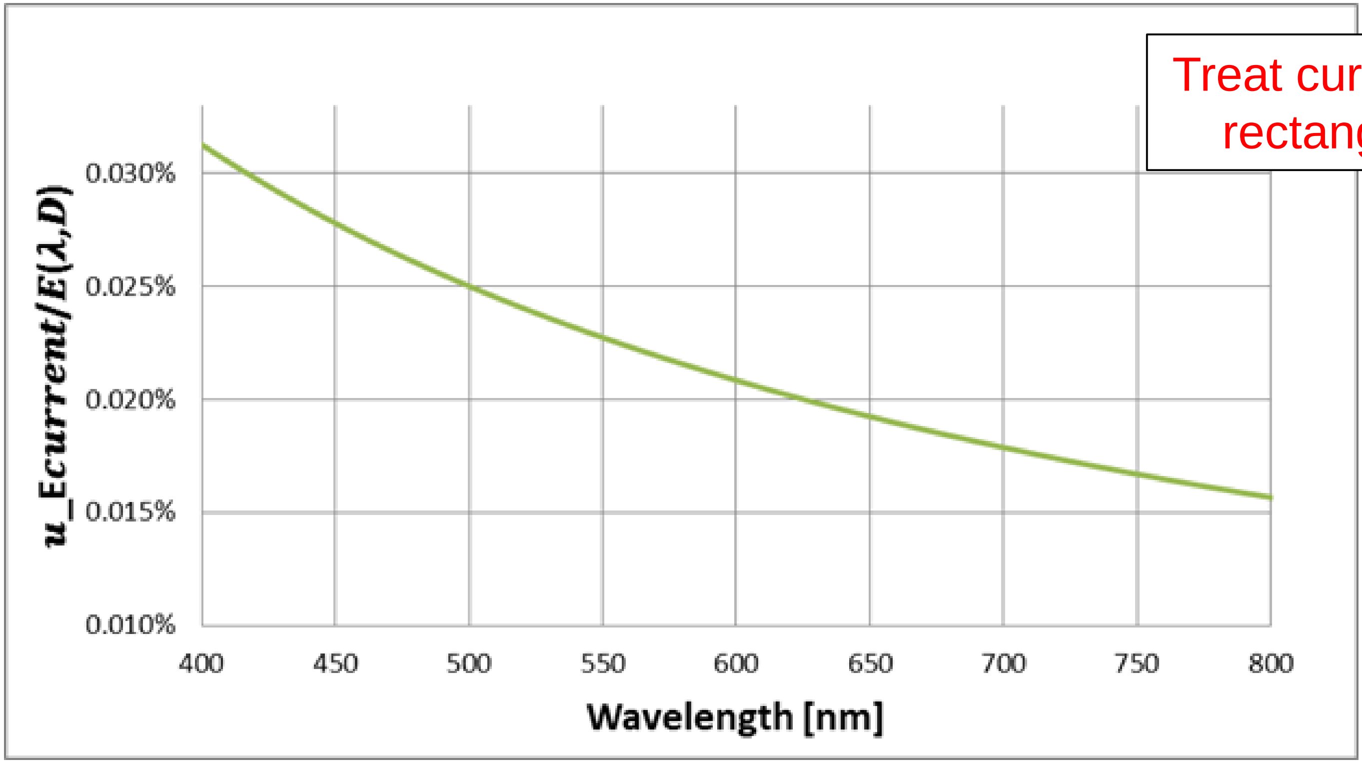
Uncertainty of additional effects (8)

Relationship between lamp current and irradiance **modelled** by:

1. Consider effect of change in current on lamp power ($P_{elec} \propto I^2$)
2. Assume direct relationship between lamp electrical and optical power
3. Assume lamp behaves similarly to blackbody radiator for small changes in power i.e. relationship between power and temperature is $P_{opt} \propto T^4$
4. So $I^2 \propto T^4$ or $T \propto I^{0.5}$. Relative change in T is half relative change in I
5. Determine spectral change in irradiance for BB associated with change in T



Treat current uncertainty as rectangular distribution

Lamp current effect by modelling

- Suppose:
  - Lamp current uncertainty is 0.020 A
  - Lamp current is 8.000 A
  - Lamp CCT is 3000 K
- Therefore, by **modelling**:
  - $u(I)$ = 0.25 %
  - $u(T)$ = 0.125 % (relative change in T is half relative change in I)
  - Absolute uncertainty in T at 3000 K = 3.8 K
  - Using Planck equation:
    - Uncertainty in spectral irradiance is 1.70 % at 350 nm
    - Uncertainty in spectral irradiance is 0.99 % at 600 nm

Lamp current effect by measurement

- Suppose:
  - Lamp current uncertainty is 0.020 A
  - Lamp current is 8.000 A
  - Lamp CCT is 3000 K

- We can **measure** the effect by:
  - Measuring irradiance at a current of 8.000 A
  - Measuring irradiance at a current of (say) 7.800 A (needs to be large enough difference to enable effect to be measured reliably)
  - $u(E_{\text{current}})$ = 0.1 x observed relative change in irradiance due to 0.200 A change in current

More details in course notes

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Uncertainty of additional effects (9)

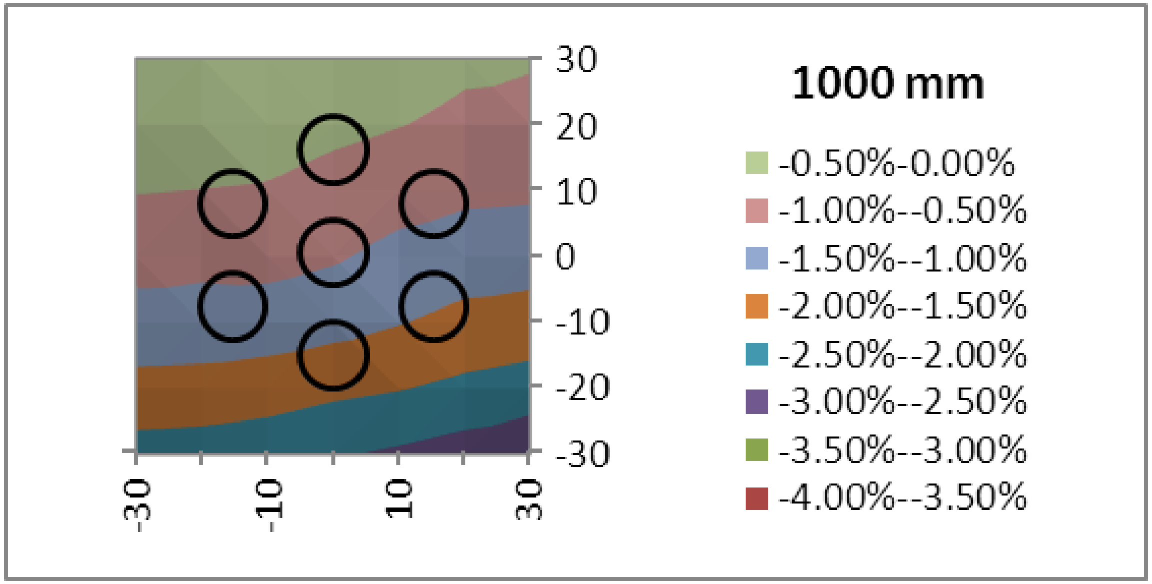
$$L_s = \frac{E_{\mathrm{FEL}}\beta_{0:45}}{\pi}\frac{d_{\mathrm{cal}}^2}{d_{\mathrm{use}}^2} K_{\mathrm{lamp\_stab}} K_{\mathrm{align}} K_{\mathrm{current}} K_{\mathrm{diff\_stab}} K_{\mathrm{unif}}$$

By combination of measurement and ‘modelling’

$$V_S = V_{\mathrm{light}} K_{\mathrm{light\_stab}} + K_{\mathrm{stray}} - V_{\mathrm{dark}} K_{\mathrm{dark\_stab}}$$

Uncertainty of additional effects (10)

Measured uniformity and consideration of where radiometer is placed



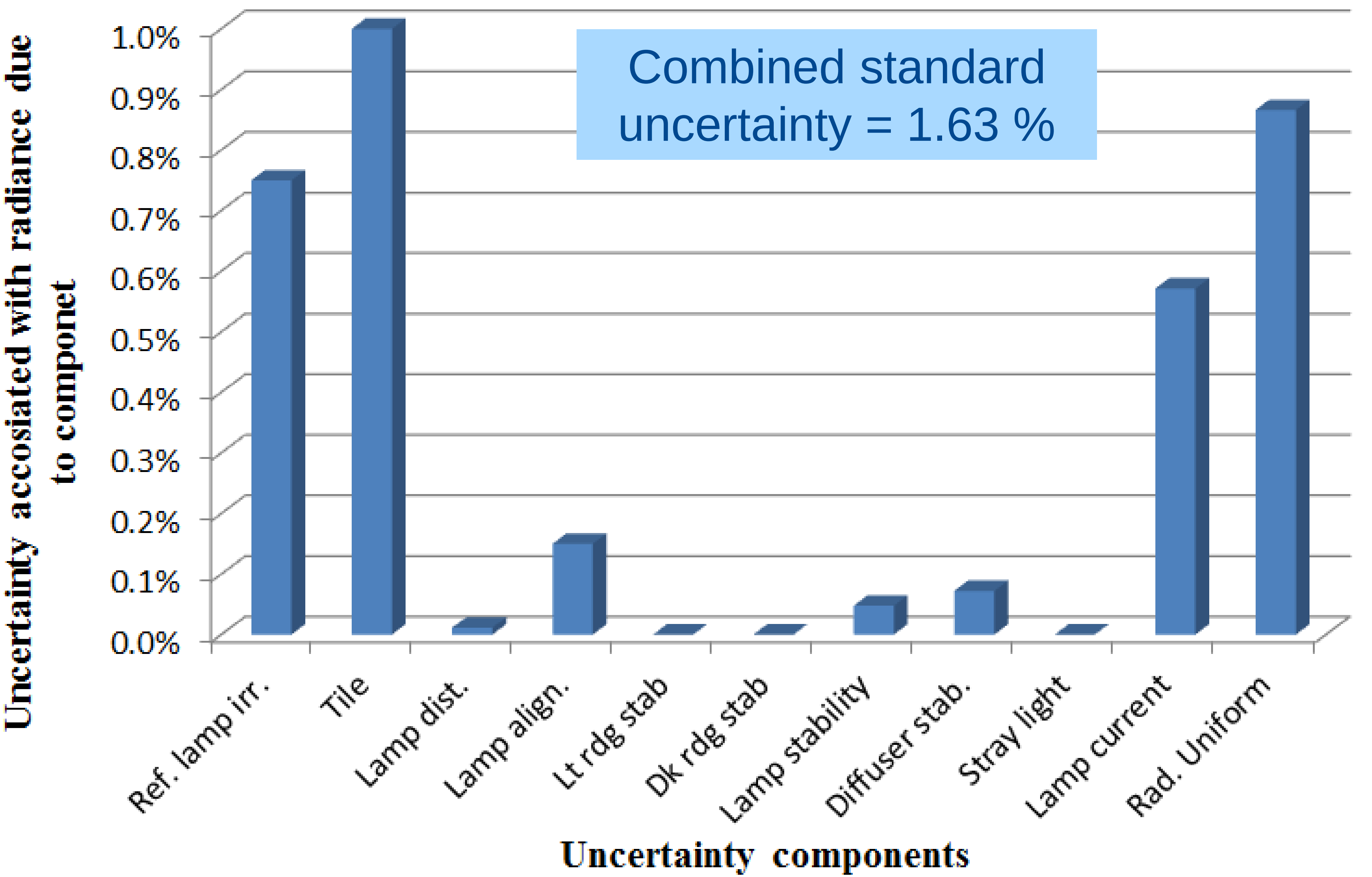
Worst case uncertainty is ± 1.5 %

Even if diffuser is perfectly uniform, non-uniformity due to lamp irradiance can be large, especially at short distances (inverse square law)

Combined uncertainty

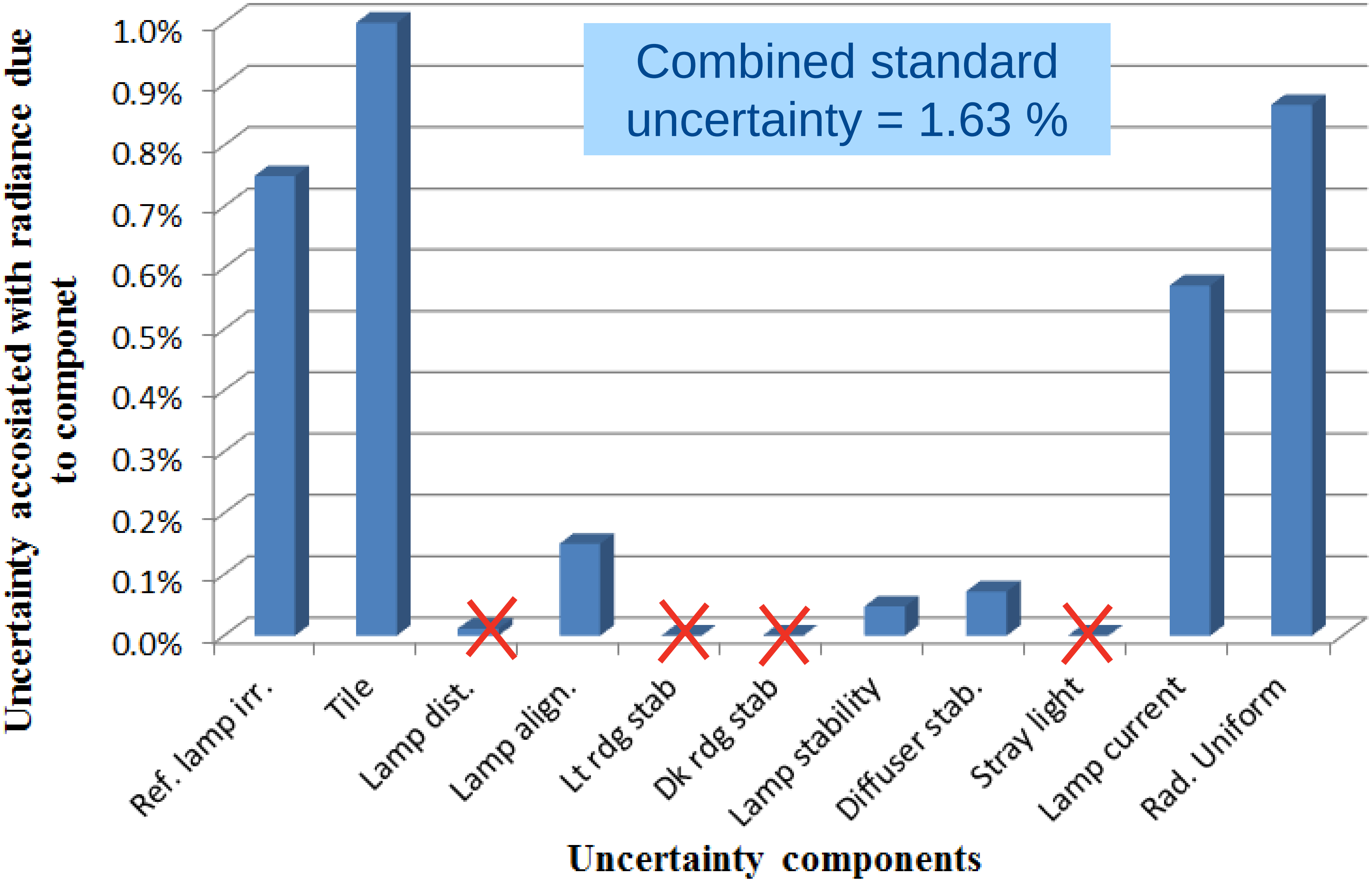
Symbol	Uncertainty component	Size of effect	Correction applied?	Residual uncertainty	Divisor	Sensitivity coefficient	Uncertainty associated with final value due to effect
$u(E_{\mathrm{FEL}})$	Ref. lamp irradiance	1.5 %	N	1.5 %	2	1	0.75 %
$u(\beta_{0:45})$	Tile radiance factor	2.0 %	N	2.0 %	2	1	1.00 %
$u(d_{\mathrm{use}})$	Lamp distance (500 mm)	0.05 mm	N	0.01 %	√3	2	0.012 %
$u(K_{\mathrm{align}})$	Lamp alignment	0.15 %	N	0.15 %	1	1	0.15 %
$u(K_{\mathrm{l\_stab}})$	Light reading stability	negligible	N	negligible			negligible
$u(K_{\mathrm{d\_stab}})$	Dark reading stability	negligible	N	negligible			negligible
$u(K_{\mathrm{lamp\_stab}})$	Lamp stability	0.083 %	N	0.083 %	√3	1	0.048 %
$u(K_{\mathrm{diff\_stab}})$	Diffuser stability	0.125 %	N	0.125 %	√3	1	0.072 %
$u(K_{\mathrm{stray}})$	Stray light in lab	negligible	N	negligible			negligible
$u(K_{\mathrm{current}})$	Lamp current (8.000 A)	0.004 A	N	0.25 % in I, or 0.99 % in E_{FEL} at 600 nm	√3	1	0.572 % (at 600 nm)
$u(K_{\mathrm{unif}})$	Radiance uniformity	1.50 %	N	1.50 %	√3	1	0.866 %
Combined standard uncertainty							1.63 %
Expanded uncertainty (*k*=2)							3.3 %

When to stop



Combined standard uncertainty = 1.63 %
Uncertainty accosiated with radiance due to componet
1.0%
0.9%
0.8%
0.7%
0.6%
0.5%
0.4%
0.3%
0.2%
0.1%
0.0%
Ref. lamp irr.
Tile
Lamp dist.
Lamp align.
Lt rdg stab
Dk rdg stab
Lamp stability
Diffuser stab.
Stray light
Lamp current
Rad. Uniform
Uncertainty components

When to stop



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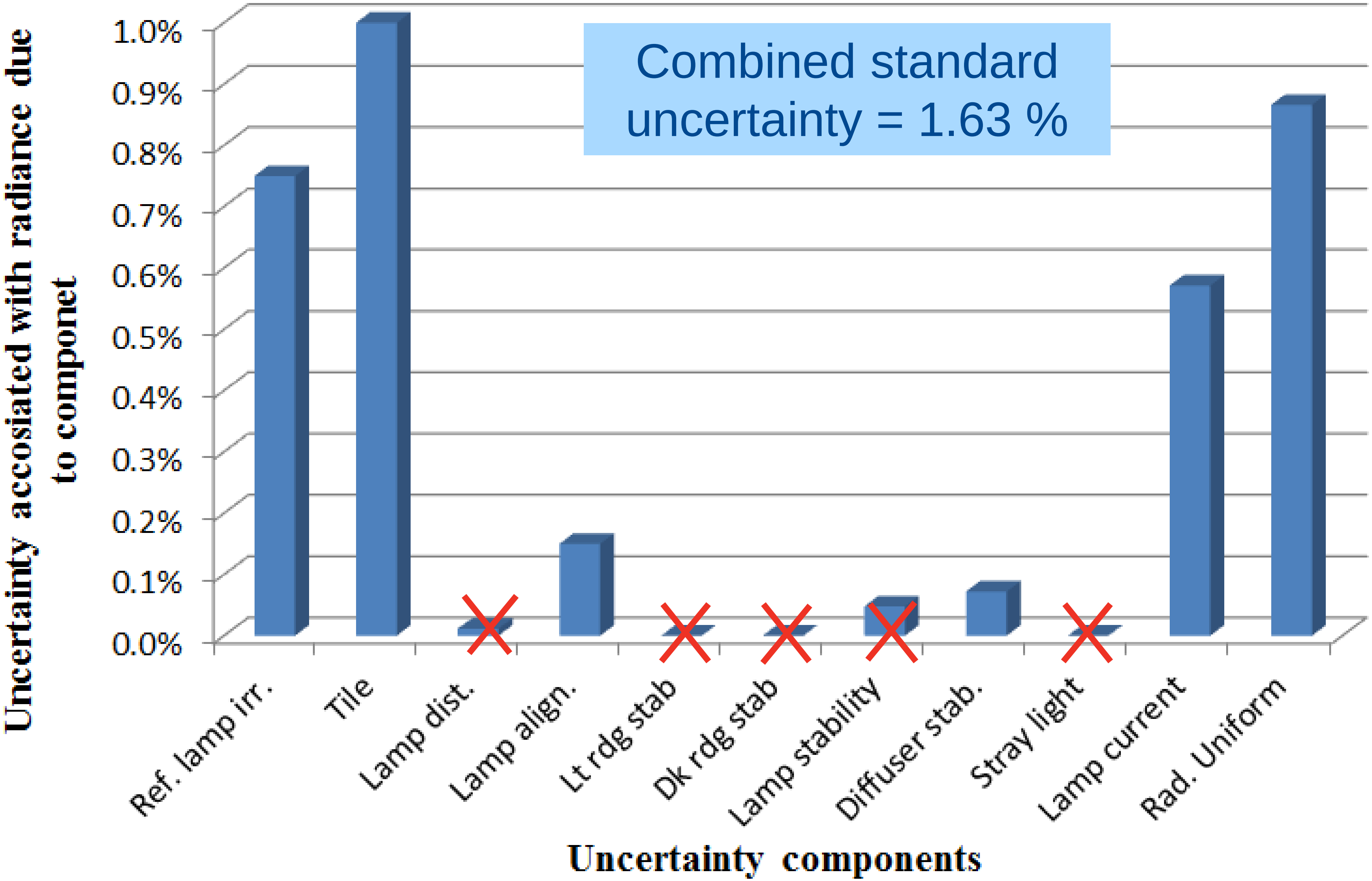
When to stop



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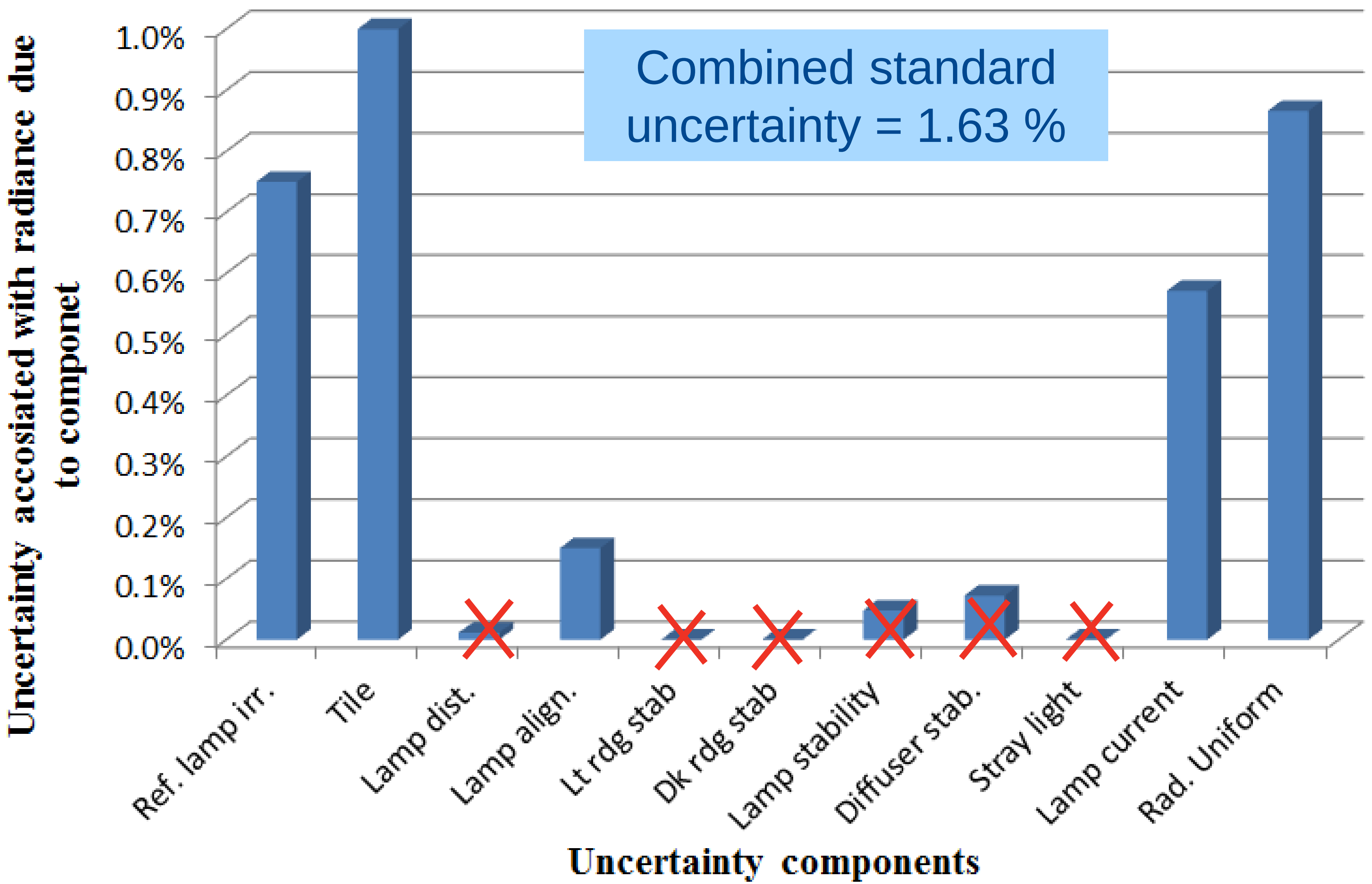
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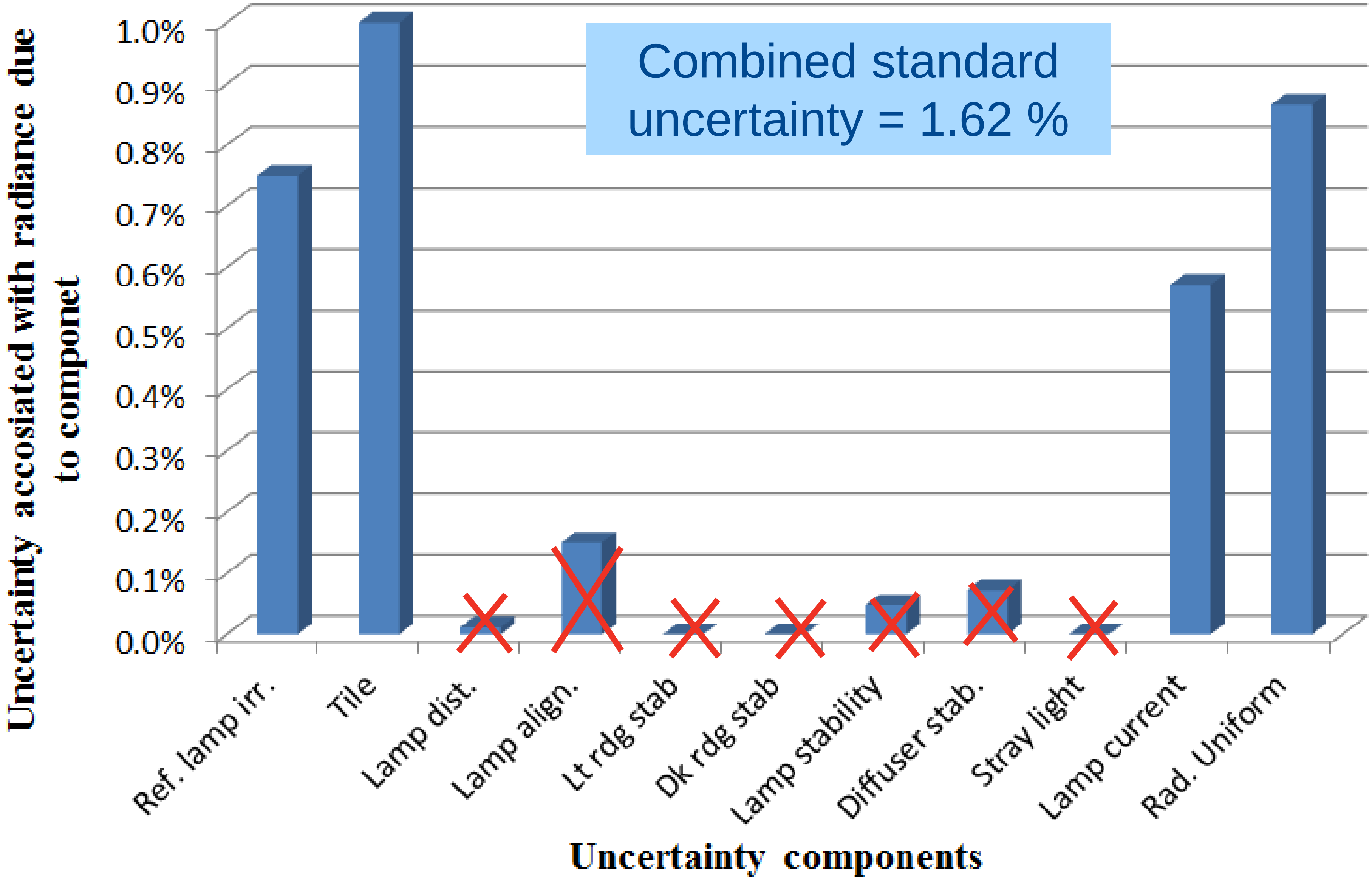

Combined standard uncertainty = 1.62 %
Uncertainty accosiated with radiance due to componet
1.0%
0.9%
0.8%
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Laboratory based

Integrating sphere

RADIANCE CALIBRATION 2

National Physical Laboratory

Training

At the end of this module, you should understand

- The difference between calculation and measurement equations
  - Why the measurement equation has more components
  - Where do they come from

Integrating sphere

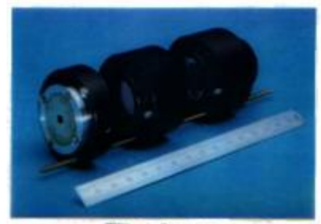
- How to develop a measurement equation
  - How to define additional components in the measurements equation
  - When to stop adding them

- How to determine the sensitivity coefficients
  - Analytically
  - Modelling
  - Lab experiment

Traceability chain

SI

Cryogenic Radiometry



Filter radiometer

Transfer radiometer

Sphere

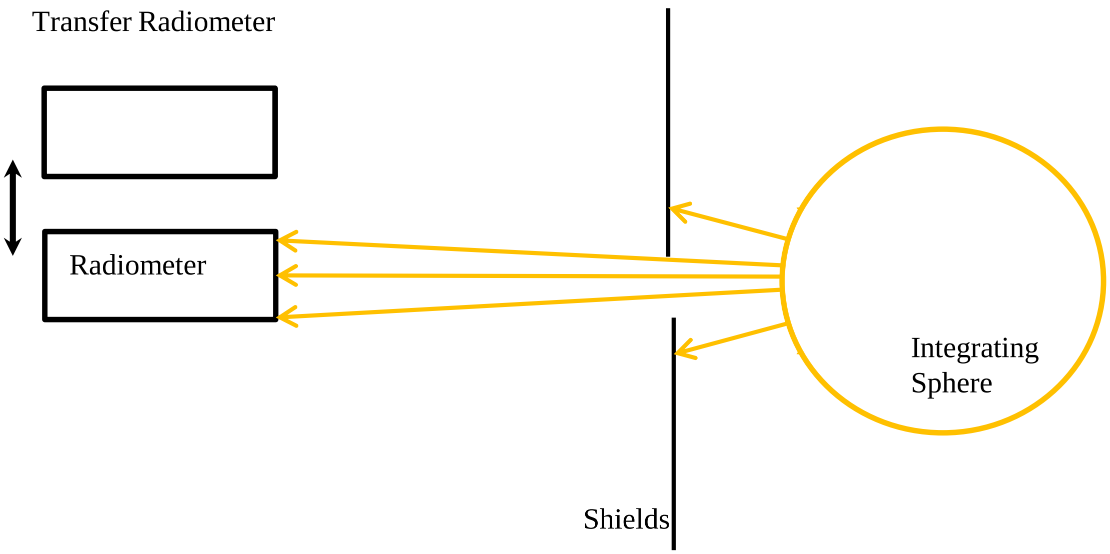
Instrument

Calculation equations

Absolute calibration coefficient

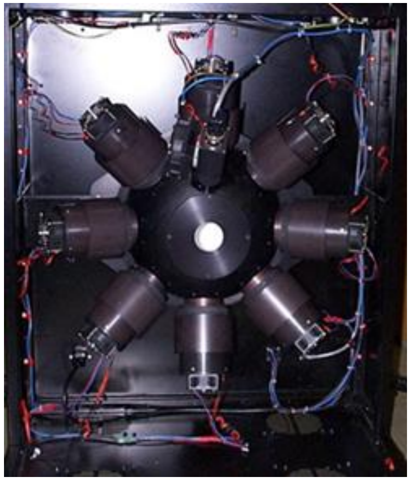
$$A_L = \frac{L_s}{V_s}$$

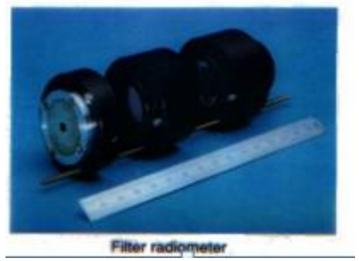
$$L_s = V_{TR} G_{TR}$$



$$V_S = V_{light} - V_{dark}$$

Sources of uncertainty





Calibration certificate

Random noise

Instrument additional effects

- Stability (drift)
- Room stray light
- Environmental sensitivity

Random noise

Instrument additional effects

- Stability (drift)
- Room stray light

Sphere effects

- Ageing
- Uniformity
- Stability
- Back reflection
- Environmental sensitivity

Measurement equations, sensitivity coefficients and assigning uncertainties

$$V_{\mathrm{S}} = V_{\mathrm{light}} - V_{\mathrm{dark}}$$

$$V_{\mathrm{S}} = V_{\mathrm{light}} K_{\mathrm{light\_stab}} + K_{\mathrm{stray}} - V_{\mathrm{dark}} K_{\mathrm{dark\_stab}}$$

Measurement equations, sensitivity coefficients and assigning uncertainties

$$V_S = V_{light} - V_{dark}$$

$$V_S = V_{light} K_{light\_stab} + K_{stray} - V_{dark} K_{dark\_stab}$$

Statistical analysis of repeated measurements

Measurement equations, sensitivity coefficients and assigning uncertainties

$$V_S = V_{light} - V_{dark}$$

$$V_S = V_{light} K_{light\_stab} + K_{stray} - V_{dark} K_{dark\_stab}$$

By measurement, using a small baffle to prevent direct radiation from sphere reaching detector

Measurement equations, sensitivity coefficients and assigning uncertainties

$$\boxed{L_{\mathrm{s}} = V_{\mathrm{TR}} G_{\mathrm{TR}}}$$

$$L_{\mathrm{s}} = (V_{\mathrm{TR\_light}} - V_{\mathrm{TR\_dark}}) G_{\mathrm{TR}} K_{\mathrm{TR\_dft}} K_{\mathrm{stray}} K_{\mathrm{temp}} K_{\mathrm{lin}}$$

$$K_{\mathrm{sph\_stab}} K_{\mathrm{sph\_temp}} K_{\mathrm{reflect}} K_{\mathrm{sph\_age}}$$

Measurement equations, sensitivity coefficients and assigning uncertainties

$$L_s = V_{TR} G_{TR}$$

$$L_s = (V_{TR\_light} - V_{TR\_dark}) G_{TR} K_{TR\_dft} K_{stray} K_{temp} K_{lin} K_{sph\_stab} K_{sph\_temp} K_{reflect} K_{sph\_age}$$

Uncertainty due to noise on these evaluated by statistical analysis of repeated measurements

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Measurement equations, sensitivity coefficients and assigning uncertainties

$$\boxed{L_s = V_{TR} G_{TR}}$$

$$L_s = (V_{TR\_light} - V_{TR\_dark}) G_{TR} K_{TR\_dft} K_{stray} K_{temp} K_{lin}$$
$$K_{sph\_stab} K_{sph\_temp} K_{reflect} K_{sph\_age}$$

Stability of sphere during measurements; included in measurement of noise on detector signals and therefore taken as zero to avoid 'double counting'

Measurement equations, sensitivity coefficients and assigning uncertainties

$$L_s = V_{TR} G_{TR}$$

$$L_s = (V_{TR\_light} - V_{TR\_dark}) G_{TR} K_{TR\_dft} K_{stray} K_{temp} K_{lin} K_{sph\_stab} K_{sph\_temp} K_{reflect} K_{sph\_age}$$

From calibration certificate

Measurement equations, sensitivity coefficients and assigning uncertainties

$$L_s = V_{TR} G_{TR}$$

$$L_s = (V_{TR\_light} - V_{TR\_dark}) G_{TR} K_{TR\_dft} K_{stray} K_{temp} K_{lin} K_{sph\_stab} K_{sph\_temp} K_{reflect} K_{sph\_age}$$

Historical calibration records; data from other researchers

Measurement equations, sensitivity coefficients and assigning uncertainties

$$L_s = V_{TR} G_{TR}$$

$$L_s = (V_{TR\_light} - V_{TR\_dark}) G_{TR} K_{TR\_dft} K_{stray} K_{temp} K_{lin}$$
$$K_{sph\_stab} K_{sph\_temp} K_{reflect} K_{sph\_age}$$

By measurement e.g. by systematic investigation of change in signal as temperature of transfer radiometer or sphere is changed

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Measurement equations, sensitivity coefficients and assigning uncertainties

$$L_s = V_{TR} G_{TR}$$

$$L_s = (V_{TR\_light} - V_{TR\_dark}) G_{TR} K_{TR\_dft} K_{stray} K_{temp} K_{lin} K_{sph\_stab} K_{sph\_temp} K_{reflect} K_{sph\_age}$$

By modelling or from repeated measurements using reference detector

e.g. experimental investigation of change in reflectance of sphere coating following exposure to sphere light source, 'sphere equation' relating reflectance to sphere throughput, and time for which lamps have been operated between calibration with transfer radiometer and measurement with test radiometer

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Combined uncertainty

$$A_{\mathrm{L}} = \frac{L_{\mathrm{s}}}{V_{\mathrm{s}}}$$

$$\left(\frac{u(L_{\mathrm{s}})}{L_{\mathrm{s}}}\right)^2 = \frac{\left(u^2\left(V_{\mathrm{TR\_light}}\right) + u^2\left(V_{\mathrm{TR\_dark}}\right)\right)}{\left(V_{\mathrm{TR\_light}} - V_{\mathrm{TR\_dark}}\right)^2}\left(\frac{u(G_{\mathrm{TR}})}{G_{\mathrm{TR}}}\right)^2 +$$

$$u^2\left(K_{\mathrm{TR\_dft}}\right) + u^2\left(K_{\mathrm{stray}}\right) + u^2\left(K_{\mathrm{temp}}\right) + u^2\left(K_{\mathrm{lin}}\right) +$$

$$u^2\left(K_{\mathrm{sph\_temp}}\right) + u^2\left(K_{\mathrm{reflect}}\right) + u^2\left(K_{\mathrm{sph\_age}}\right) + u^2\left(K_{\mathrm{sph\_stab}}\right)$$

Note: in this analysis, assume that uncertainties due to noise on readings and stray are negligible for test radiometer, so relative uncertainty associated with A_{L} is same as that associated with L_{S}

Combined uncertainty

Symbol	Uncertainty component	Size of effect	Correction applied?	Residual uncertainty	Divisor	Sensitivity coefficient	Uncertainty associated with final value due to effect
$u(V_{\mathrm{TR\_light}})$	Transfer radiometer light rdg	0.001 %	N	0.001 %	1	1	0.001 %
$u(V_{\mathrm{TR\_dark}})$	Transfer radiometer light rdg	0.001 %	N	0.001 %	1	1	0.001 %
$u(G_{\mathrm{TR}})$	Transfer radiometer gain	3.0 %	N	3.0 %	2	1	1.50 %
$u(K_{\mathrm{TR\_drift}})$	Transfer radiometer change since calibration	0.5 %	N	0.5 %	√3	1	0.289 %
$u(K_{\mathrm{stray}})$	Stray light with transfer radiometer	negligible	N	negligible			negligible
$u(K_{\mathrm{temp}})$	Temperature effects for transfer radiometer	negligible	N	negligible			negligible
$u(K_{\mathrm{lin}})$	Transfer radiometer linearity	0.083 %	N	0.083 %	√3	1	0.048 %
$u(K_{\mathrm{sph\_temp}})$	Temperature effects for sphere	negligible	N	negligible			negligible
$u(K_{\mathrm{reflect}})$	Inter-reflections between radiometers and sphere	0.125 %	N	0.125 %	√3	1	0.072 %
$u(K_{\mathrm{sph\_age}})$	Ageing of sphere	0.125 %	N	0.125 %	√3	1	0.072 %
$u(K_{\mathrm{sph\_stab}})$	Sphere stability during measurements	Included in $u(V_{\mathrm{TR\_light}})$ and $u(V_{\mathrm{TR\_dark}})$	N	0 %	1	1	0 %
Combined standard uncertainty							1.53 %
Expanded uncertainty (*k*=2)							3.1 %

Note: in this analysis, assume that uncertainties due to noise on readings and stray are negligible for test radiometer, so relative uncertainty associated with A_{L} is same as that associated with L_{S}

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Combined uncertainty

Symbol	Uncertainty component	Size of effect	Correction applied?	Residual uncertainty	Divisor	Sensitivity coefficient	Uncertainty associated with final value due to effect
$u(V_{\mathrm{TR\_light}})$	Transfer radiometer light rdg	0.001 %	N	0.001 %	1	1	0.001 %
$u(V_{\mathrm{TR\_dark}})$	Transfer radiometer light rdg	0.001 %	N	0.001 %	1	1	0.001 %
$u(G_{\mathrm{TR}})$	Transfer radiometer gain	3.0 %	N	3.0 %	2	1	1.50 %
$u(K_{\mathrm{TR\_drift}})$	Transfer radiometer change since calibration	0.5 %	N	0.5 %	√3	1	0.289 %
$u(K_{\mathrm{stray}})$	Stray light with transfer radiometer	negligible	N	negligible			negligible
$u(K_{\mathrm{temp}})$	Temperature effects for transfer radiometer	negligible	N	negligible			negligible
$u(K_{\mathrm{lin}})$	Transfer radiometer linearity	0.083 %	N	0.083 %	√3	1	0.048 %
$u(K_{\mathrm{sph\_temp}})$	Temperature effects for sphere	negligible	N	negligible			negligible
$u(K_{\mathrm{reflect}})$	Inter-reflections between radiometers and sphere	0.125 %	N	0.125 %	√3	1	0.072 %
$u(K_{\mathrm{sph\_age}})$	Ageing of sphere	0.125 %	N	0.125 %	√3	1	0.072 %
$u(K_{\mathrm{sph\_stab}})$	Sphere stability during measurements	Included in $u(V_{\mathrm{TR\_light}})$ and $u(V_{\mathrm{TR\_dark}})$	N	0 %	1	1	0 %
Combined standard uncertainty							1.53 %
Expanded uncertainty (*k*=2)							3.1 %

Note: in this analysis, assume that uncertainties due to noise on readings and stray are negligible for test detector, so relative uncertainty associated with A_{L} is same as that associated with L_{S}

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CONCLUSIONS

Conclusions

1. Traceability Chain
   - Show linkage back to 'point of trust'
2. Calculation Equation
   - Equation for each step in measurement process / step in chain
3. Sources of Uncertainty
   - Consider all factors that may affect result for each calculation equation
4. Measurement Equation
   - Include all sources of uncertainty
   - Types of uncertainty (multiplicative, additive)
5. Sensitivity Coefficients
   - Mathematically, from experimental investigations, or by modelling
6. Assigning Uncertainties
   - Other information (e.g. certificates, historical data, other researchers), statistical analysis, experimental studies, modelling, or combination
7. Combining your uncertainties
   - When to stop
8. Expanding your uncertainties