Introduction to radiometric uncertainty

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About NPL …

• Founded in 1900
• World leading National Metrology Institute
• ~750 staff; 550+ specialists in Measurement Science plus 200 visiting researchers pa
• State-of-the-art laboratory facilities
• 388 Laboratories (35,746 sq. metres)
• The heart of the UK’s National Measurement System to support business and society
• Experts in Knowledge Transfer
The growing demand for better measurements

Health & safety  Environment  Healthcare  Manufacturing

Energy  Communications  Transport  Science

2% of GDP dependent on a robust measurement system
<table>
<thead>
<tr>
<th>SI Units</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>kg</td>
<td>kilogram</td>
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<tr>
<td>m</td>
<td>metre</td>
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<td>s</td>
<td>second</td>
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<td>A</td>
<td>ampere</td>
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<td>K</td>
<td>kelvin</td>
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<td>cd</td>
<td>candela</td>
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<td>mol</td>
<td>mole</td>
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<td></td>
<td>Mass</td>
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<td></td>
<td>Length</td>
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<td></td>
<td>Time</td>
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<td></td>
<td>Electric Current</td>
</tr>
<tr>
<td></td>
<td>Temperature</td>
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<tr>
<td></td>
<td>Luminous Intensity</td>
</tr>
<tr>
<td></td>
<td>Amount of Substance</td>
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</tbody>
</table>
KC: Luminous intensity

Source: BIPM Key Comparison database (http://kcdb.bipm.org/)
The Guide to the expression of Uncertainty in Measurement (GUM)

The foremost authority and guide to the expression and calculation of uncertainty in measurement science
Written by the JCGM and BIPM
Covers a wide number of applications
Technical with formal mathematics

**Uncertainty** – Error – Correction

**Uncertainty**
Describes the spread
Drawn from a probability distribution described by uncertainty

**Error**
Difference to the (unknown) true value
Residual, uncorrectable, unknown error

**Correction**
Known offset from true value
Traceability

“Property of a measurement result relating the result to a stated metrological reference (free definition and not necessarily SI) through an unbroken chain of calibrations of a measuring system or comparisons, each contributing to the stated measurement uncertainty”

Committee on Earth Observation Satellites (CEOS)
Spectral Radiance and Irradiance

SI Units

Cryogenic radiometer

Primary Standard

Laser

Reference photodiode

Radiance (T via Planck)

Spectrometer Radiance / Irradiance

Filter-radiometer

Blackbody 3500 K

Satellite Earth Imager

Standard lamp
Traceability: further points

- Cryogenic radiometer 0.01%
- Primary irradiance standard 0.5%
- Calibration lamp use ‘in situ’ 1.2%
- Field spectrometer calibration 2.5%
- Vicarious calibration reference 3.2%
Inverse Square Law of Irradiance

\[ E = \frac{\Phi}{A} = \frac{I}{d^2} \]

E Irradiance
\( \text{W m}^{-2} \)

I Intensity
\( \text{W sr}^{-1} \)

Palmer J.M, 2010
Irradiance Standards

- Tungsten-halogen lamp (FEL) 1 kW (~3000 K)
Irradiance

\[ E(\lambda, d) = E(\lambda, 500 \text{ mm}) \left( \frac{500 \text{ mm}}{d} \right)^2 \]
Radiance invariance

- Throughput invariance (étendue) $T = A\Omega$

Assuming lossless beam propagation and no lens transmission

Palmer J.M, 2010
Radiance standards

- Lamp – reflectance standard
- Integrating sphere
Radiance

Lamp - tile

\[ L_s = \frac{E_{FEL} \beta_{0:45}}{\pi} \frac{d_{cal}^2}{d_{use}^2} \]
Reflectance $\rho$ and Reflectance factors

We have 9 kinds of reflectance and 9 equivalent reflectance factors.
First defined in 1977 by Nicodemous to simply surface scattering phenomena,

Assumptions:
Geometrical ray optics
A flat surface that is uniformly illuminated
Incident radiance depends only on direction
Surface has uniform and isotropic scattering properties

Nicodemous, 1977
BRDF bidirectional reflectance function

Definition

\[
f(\theta_1, \varphi_1; \theta_R, \varphi_R, \lambda) = \frac{dL_R(\theta_1, \varphi_1; \theta_R, \varphi_R, \lambda, E_I)}{dE_I(\theta_1, \varphi_1)}
\]

Measurement equation

\[
BRDF =
\]

\[
f_r(\theta_1, \varphi_1, \theta_R, \varphi_R, \lambda) = d\rho(\theta_1, \varphi_1, \theta_R, \varphi_R, \lambda) = \lim_{\Omega \to 0} \frac{\Phi_R(\theta_1, \varphi_1, \theta_R, \varphi_R, \lambda)}{\Phi_I(\lambda) \cos \theta_R \Omega}
\]
BRF bidirectional reflectance factor

\[
BRF(\theta_i ; \theta_r, \lambda) = BRDF \cdot \pi = \lim_{\Omega \to 0} \frac{\Phi_R(\theta_I, \theta_R, \lambda)}{\Phi_I(\lambda) \cos \theta_R \Omega} \cdot \pi
\]

the ratio of the radiance flux actually reflected by a sample surface to that which would be reflected into the same reflected-beam geometry by an ideal perfectly diffuse standard surface irradiated in exactly the same way as the sample
Radiometers

- **Multispectral**
  Stable and reliable
  Limited spectral information

- **Hyperspectral**
  Demanding characterisation necessary!
  “Full” spectral information

Ehsani et al. 1998
Steps to an uncertainty budget

1. Traceability Chain
2. Calculation Equation
3. Sources of Uncertainty
4. Measurement Equation
5. Sensitivity Coefficients
6. Assigning Uncertainties
7. Combining your uncertainties

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Uncertainty component</th>
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<th>Sensitivity coefficient</th>
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</table>

<table>
<thead>
<tr>
<th>Combined standard uncertainty</th>
<th>Expanded uncertainty</th>
</tr>
</thead>
</table>
Step 1: Traceability chain

- **SI**
  - NMI calibration of reference lamp
  - Secondary lab lamp calibration
  - Radiometer

- **Secondary lab**
  - Calibration
  - Radiometer
  - Secondary lab lamp calibration

- **NMI**
  - Calibration of reflectance panel
  - Points of trust
Step 2: Calculation equations

Absolute calibration coefficient

\[ A_L = \frac{L_s}{V_s} \]

\[ L_s = \frac{E_{FEL} \beta_{0:45}}{\pi} \frac{d^2_{cal}}{d^2_{use}} \]

\[ V_S = V_{\text{light}} - V_{\text{dark}} \]
Step 3: Sources of uncertainty

Lamp
Lamp additional effects
- Ageing
- Alignment
- Current stability

Diffuser
Diffuser additional effects
- Ageing
- Uniformity

Distance accuracy

Random noise
Instrument additional effects
- Stability (drift)
- Room stray light
$$L_s = \frac{E_{\text{FEL}} \beta_{0:45}}{\pi} \frac{d_{\text{cal}}^2}{d_{\text{use}}^2}$$

$$L_s = \frac{E_{\text{FEL}} \beta_{0:45}}{\pi} \frac{u_{\text{cal}}}{d_{\text{use}}^2} K_{\text{lampstab}} K_{\text{align}} K_{\text{current}} K_{\text{diffstab}} K_{\text{unif}}$$

$$V_S = V_{\text{light}} - V_{\text{dark}}$$

$$V_S = V_{\text{light}} K_{\text{lightstab}} + K_{\text{stray}} - V_{\text{dark}} K_{\text{darkstab}}$$
Determining the sensitivity coefficients

• Do an experiment

• Analytical expression

• Model it
## Uncertainties of calculation equation components (1)

<table>
<thead>
<tr>
<th>Component</th>
<th>Sensitivity coefficient</th>
<th>Relative radiance uncertainty due to ...</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_i$</td>
<td>$c_i = \frac{\partial f}{\partial x_i}$</td>
<td>$c_i u(x_i)$</td>
</tr>
</tbody>
</table>

- **Lamp irradiance** $E_{FEL}$  
  $L_s / E_{FEL}$  
  $1 \cdot u(E_{FEL}) / E_{FEL}$

- **Radiance factor** $\beta_{0:45}$  
  $L_s / \beta_{0:45}$  
  $1 \cdot u(\beta_{0:45}) / \beta_{0:45}$

- **Distance** $d_{use}^2$  
  $-2L_s / d_{use}$  
  $-2 \cdot u(d_{use}) / d_{use}$

\[
\left( \frac{u(L)}{L} \right)^2 = \left( \frac{u(E_{FEL})}{E_{FEL}} \right)^2 + \left( \frac{u(\beta_{0:45})}{\beta_{0:45}} \right)^2 + (-2)^2 \left( \frac{u(d_{use})}{d_{use}} \right)^2
\]  

From calibration certificates
Certificate uncertainties

From calibration certificates

\[ L_s = \frac{E_{\text{FEL}}}{\beta_{0.45}} \times \frac{d^2_{\text{cal}}}{d^2_{\text{use}}} \times K_{\text{lamp stab}} \times K_{\text{align}} \times K_{\text{current}} \times K_{\text{diff stab}} \times K_{\text{unif}} \]

Remember calibration certificates almost always quote uncertainties at \( k = 2 \)!

Certificate may not give required quantity (modelling may be needed to obtain desired quantity): read it carefully!

\[ \rho \neq \beta_{0.45} \]

Integration and/or interpolation may need to be done if the wavelength values or radiometer bands do not match.
### Uncertainties of calculation equation components - sensitivity

<table>
<thead>
<tr>
<th>Component</th>
<th>Sensitivity coefficient $c_i = \frac{\partial f}{\partial x_i}$</th>
<th>Relative radiance uncertainty due to ... $c_i \mu(x_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lamp irradiance $E_{\text{FEL}}$</td>
<td>$L_s / E_{\text{FEL}}$</td>
<td>$1 \cdot u(E_{\text{FEL}}) / E_{\text{FEL}}$</td>
</tr>
<tr>
<td>Radiance factor $\beta_{0:45}$</td>
<td>$L_s / \beta_{0:45}$</td>
<td>$1 \cdot u(\beta_{0:45}) / \beta_{0:45}$</td>
</tr>
<tr>
<td>Distance $d_{\text{use}}^2$</td>
<td>$-2L_s / d_{\text{use}}$</td>
<td>$-2 \cdot u(d_{\text{use}}) / d_{\text{use}}$</td>
</tr>
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</table>

\[
\left( \frac{u(L)}{L} \right)^2 = \left( \frac{u(E_{\text{FEL}})}{E_{\text{FEL}}} \right)^2 + \left( \frac{u(\beta_{0:45})}{\beta_{0:45}} \right)^2 + (-2)^2 \left( \frac{u(d_{\text{use}})}{d_{\text{use}}} \right)^2
\]
Rectangular uncertainty distributions

Resolution of distance measuring instrument = 0.1 mm

Measurement distance = 500.0 mm

Uncertainty associated with distance measurement = \(\frac{0.05}{500} / \sqrt{3} = 0.006\%\)

Uncertainty in irradiance from distance measurement = \(2 \times 0.006\% = 0.012\%\)
Uncertainties of calculation equation components

\[
\left( \frac{u(L)}{L} \right)^2 = \left( \frac{u(E_{FEL})}{E_{FEL}} \right)^2 + \left( \frac{u(\beta_{0.45})}{\beta_{0.45}} \right)^2 + \left( -2 \right)^2 \left( \frac{u(d_{use})}{d_{use}} \right)^2
\]

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<tbody>
<tr>
<td>( u(E_{FEL}) )</td>
<td>Ref. lamp irradiance</td>
<td>1.5 %</td>
<td>N</td>
<td>1.5 %</td>
<td>2</td>
<td>1</td>
<td>0.75 %</td>
</tr>
<tr>
<td>( u(\beta_{0.45}) )</td>
<td>Tile radiance factor</td>
<td>2.0 %</td>
<td>N</td>
<td>2.0 %</td>
<td>2</td>
<td>1</td>
<td>1.00 %</td>
</tr>
<tr>
<td>( u(d_{use}) )</td>
<td>Lamp distance (500 mm)</td>
<td>0.05 mm</td>
<td>N</td>
<td>0.01 %</td>
<td>( \sqrt{3} )</td>
<td>2</td>
<td>0.012 %</td>
</tr>
<tr>
<td>( u(d_{use}) )</td>
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</tbody>
</table>

Combined standard uncertainty

Expanded uncertainty
Uncertainty of additional effects

(1)

\[ L_s = \frac{E_{\text{FEL}} \beta_{0.45}}{\pi} \frac{d_{\text{cal}}^2}{d_{\text{use}}^2} K_{\text{lamp\_stab}} K_{\text{align}} K_{\text{current}} K_{\text{diff\_stab}} K_{\text{unif}} \]

Statistical analysis of repeated measurements

\[ V_S = V_{\text{light}} K_{\text{light\_stab}} + K_{\text{stray}} - V_{\text{dark}} K_{\text{dark\_stab}} \]

Repeat measurements with realignment of the lamp
Uncertainty of additional effects (2)

- Negligible instrument drift in controlled lab environment
  - 0.7 DN change during 45 minute constant run

\[ V_S = V_{\text{light}} K_{\text{light_stab}} + K_{\text{stray}} - V_{\text{dark}} K_{\text{dark_stab}} \]
Uncertainty of additional effects (3)

\[ L_s = \frac{E_{\text{FEL}} \beta_{0.45}}{\pi} \frac{d_{\text{cal}}^2}{d_{\text{use}}^2} K_{\text{lamp stab}} K_{\text{align}} K_{\text{current}} K_{\text{diff stab}} K_{\text{unif}} \]

Historical calibration records; data from other researchers

\[ V_s = V_{\text{light}} K_{\text{light stab}} + K_{\text{stray}} - V_{\text{dark}} K_{\text{dark stab}} \]
Uncertainty of additional effects

\[ L_s = \frac{E_{\text{FEL}} \beta_{0.45}}{\pi} \frac{\langle d_{\text{cal}}^2 \rangle}{\langle d_{\text{use}}^2 \rangle} K_{\text{lamp\_stab}} K_{\text{align}} K_{\text{current}} K_{\text{diff\_stab}} K_{\text{unif}} \]

By measurement

\[ V_s = V_{\text{light}} K_{\text{light\_stab}} + K_{\text{stray}} - V_{\text{dark}} K_{\text{dark\_stab}} \]
Uncertainty of additional effects (5)

Difference between detector dark reading and measurement with detector FOV obscured smaller than standard deviation of individual dark runs

- Room stray light negligible

\[ V_S = V_{\text{light}} K_{\text{light\_stab}} + K_{\text{stray}} - V_{\text{dark}} K_{\text{dark\_stab}} \]
Uncertainty of additional effects (6)

\[ L_s = \frac{E_{\text{FEL}} \beta_{0.45}}{\pi} \frac{d_{\text{cal}}^2}{d_{\text{use}}^2} K_{\text{lamp stab}} K_{\text{align}} K_{\text{current}} K_{\text{diff stab}} K_{\text{unif}} \]

By modelling (or by measurement)

\[ V_s = V_{\text{light}} K_{\text{light stab}} + K_{\text{stray}} - V_{\text{dark}} K_{\text{dark stab}} \]
Uncertainty of additional effects (7)

Relationship between lamp current and irradiance modelled by:

1. Consider effect of change in current on lamp power ($P_{\text{elec}} \propto I^2$).
2. Assume direct relationship between lamp electrical and optical power.
3. Assume lamp behaves similarly to blackbody radiator for small changes in power and temperature.
4. So $I^2 \propto T^4$ or $T \propto I^{0.5}$.
5. Determine spectral change in irradiance for BB associated with change in $T$.

Treat current uncertainty as rectangular distribution.
Lamp current effect by measurement

- Suppose:
  - Lamp current uncertainty is 0.020 A
  - Lamp current is 8.000 A
  - Lamp CCT is 3000 K

- We can measure the effect by:
  - Measuring irradiance at a current of 8.000 A
  - Measuring irradiance at a current of (say) 7.800 A (needs to be large enough difference to enable effect to be measured reliably)

\[ u(E_{\text{current}}) = 0.1 \times \text{observed relative change in irradiance due to } 0.200 \text{ A change in current} \]
Uncertainty of additional effects

(8)

\[ L_s = \frac{E_{\text{FEL}} \beta_{0.45}}{\pi} \frac{d_{\text{cal}}^2}{d_{\text{use}}^2} K_{\text{lamp\_stab}} K_{\text{align}} K_{\text{current}} K_{\text{diff\_stab}} K_{\text{unif}} \]

By combination of measurement and ‘modelling’

\[ V_S = V_{\text{light}} K_{\text{light\_stab}} + K_{\text{stray}} - V_{\text{dark}} K_{\text{dark\_stab}} \]
Uncertainty of additional effects (9)

Measured uniformity and consideration of where radiometer is placed

Even if diffuser is perfectly uniform, non-uniformity due to lamp irradiance can be large, especially at short distances (inverse square law)

Worst case uncertainty is ± 1.5 %
## Combined uncertainty

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<td>$u(E_{REF})$</td>
<td>Ref. lamp irradiance</td>
<td>1.5 %</td>
<td>N</td>
<td>1.5 %</td>
<td>2</td>
<td>1</td>
<td>0.75 %</td>
</tr>
<tr>
<td>$u(\beta_{0.45})$</td>
<td>Tile radiance factor</td>
<td>2.0 %</td>
<td>N</td>
<td>2.0 %</td>
<td>2</td>
<td>1</td>
<td>1.00 %</td>
</tr>
<tr>
<td>$u(d_{use})$</td>
<td>Lamp distance (500 mm)</td>
<td>0.05 mm</td>
<td>N</td>
<td>0.01 %</td>
<td>$\sqrt{3}$</td>
<td>2</td>
<td>0.012 %</td>
</tr>
<tr>
<td>$u(K_{align})$</td>
<td>Lamp alignment</td>
<td>0.15 %</td>
<td>N</td>
<td>0.15 %</td>
<td>1</td>
<td>1</td>
<td>0.15 %</td>
</tr>
<tr>
<td>$u(K_{Lstab})$</td>
<td>Light reading stability</td>
<td>negligible</td>
<td>N</td>
<td>negligible</td>
<td></td>
<td></td>
<td>negligible</td>
</tr>
<tr>
<td>$u(K_{d_stab})$</td>
<td>Dark reading stability</td>
<td>negligible</td>
<td>N</td>
<td>negligible</td>
<td></td>
<td></td>
<td>negligible</td>
</tr>
<tr>
<td>$u(K_{lamp_stab})$</td>
<td>Lamp stability</td>
<td>0.083 %</td>
<td>N</td>
<td>0.083 %</td>
<td>$\sqrt{3}$</td>
<td>1</td>
<td>0.048 %</td>
</tr>
<tr>
<td>$u(K_{diff_stab})$</td>
<td>Diffuser stability</td>
<td>0.125 %</td>
<td>N</td>
<td>0.125 %</td>
<td>$\sqrt{3}$</td>
<td>1</td>
<td>0.072 %</td>
</tr>
<tr>
<td>$u(K_{stray})$</td>
<td>Stray light in lab</td>
<td>negligible</td>
<td>N</td>
<td>negligible</td>
<td></td>
<td></td>
<td>negligible</td>
</tr>
<tr>
<td>$u(K_{current})$</td>
<td>Lamp current (8.000 A)</td>
<td>0.004 A</td>
<td>N</td>
<td>0.25 % in $I$, or 0.99 % in $E_{REF}$ at 600 nm</td>
<td>$\sqrt{3}$</td>
<td>1</td>
<td>0.572 % (at 600 nm)</td>
</tr>
<tr>
<td>$u(K_{unif})$</td>
<td>Radiance uniformity</td>
<td>1.50 %</td>
<td>N</td>
<td>1.50 %</td>
<td>$\sqrt{3}$</td>
<td>1</td>
<td>0.866 %</td>
</tr>
</tbody>
</table>

**Combined standard uncertainty**

1.63 %

**Expanded uncertainty ($k=2$)**

3.3 %
Expanding uncertainties

If the distribution is not Gaussian, then a different coverage factor is needed.
Conclusions

1. Traceability Chain
   Show linkage back to ‘point of trust’

2. Calculation Equation
   Equation for each step in measurement process / step in chain

3. Sources of Uncertainty
   Consider all factors that may affect result for each calculation equation

4. Measurement Equation
   Include all sources of uncertainty
   Types of uncertainty (multiplicative, additive)
5. Sensitivity Coefficients

Mathematically, from experimental investigations, or by modelling

6. Assigning Uncertainties

Other information (e.g. certificates, historical data, other researchers), statistical analysis, experimental studies, modelling, or combination

7. Combining your uncertainties

When to stop

8. Expanding your uncertainties
Summary

Metrology view

- SI traceability – ensures the valid measurements
- Calibration links instrument output readings to physical values
- SI traceability especially important to radiometry, as these measurements are used calibration and validation of satellite sensors
- Instrument characteristics influence their properties and performance
Thank you

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