Introduction to radiometric uncertainty

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Metrology for Earth Observation and Climate
http://www.emceoc.org

EMRP
European Metrology Research Programme
Programme of EURAMET

The EMRP is jointly funded by the EMRP participating countries within EURAMET and the European Union

EURAMET
European Association of National Metrology Institutes

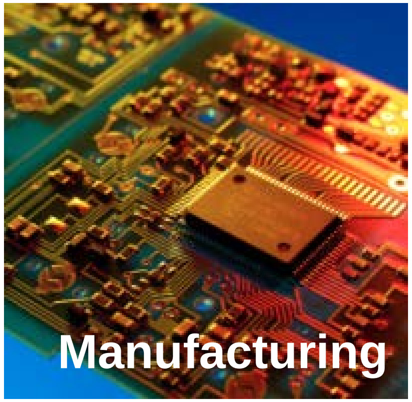
fiducial reference measurements for satellite ocean colour

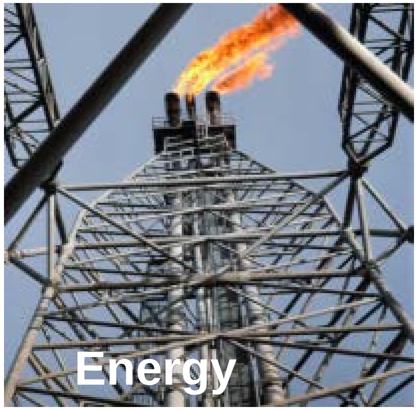
About NPL …

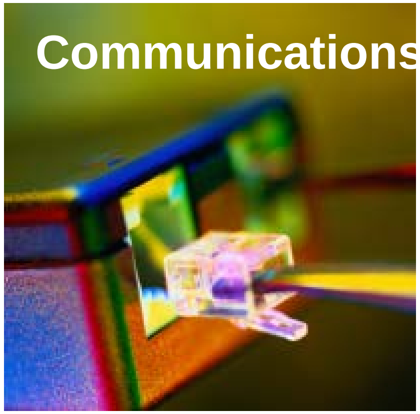
- Founded in **1900**
- World leading **National Metrology Institute**
- ~750 staff; 550+ specialists in Measurement Science plus 200 visiting researchers pa
- State-of-the-art laboratory facilities
- 388 Laboratories (35,746 sq. metres)
- The heart of the UK's **National Measurement System** to support business and society
- Experts in **Knowledge Transfer**

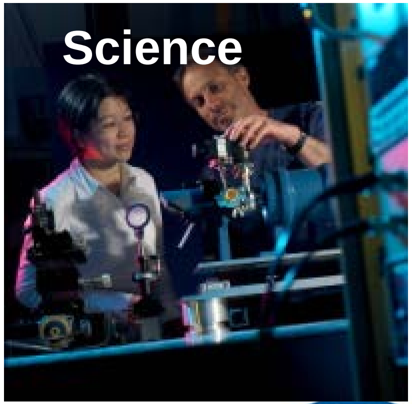
NPL National Physical Laboratory Training

The growing demand for better measurements









2% of GDP dependent on a robust measurement system

SI Units

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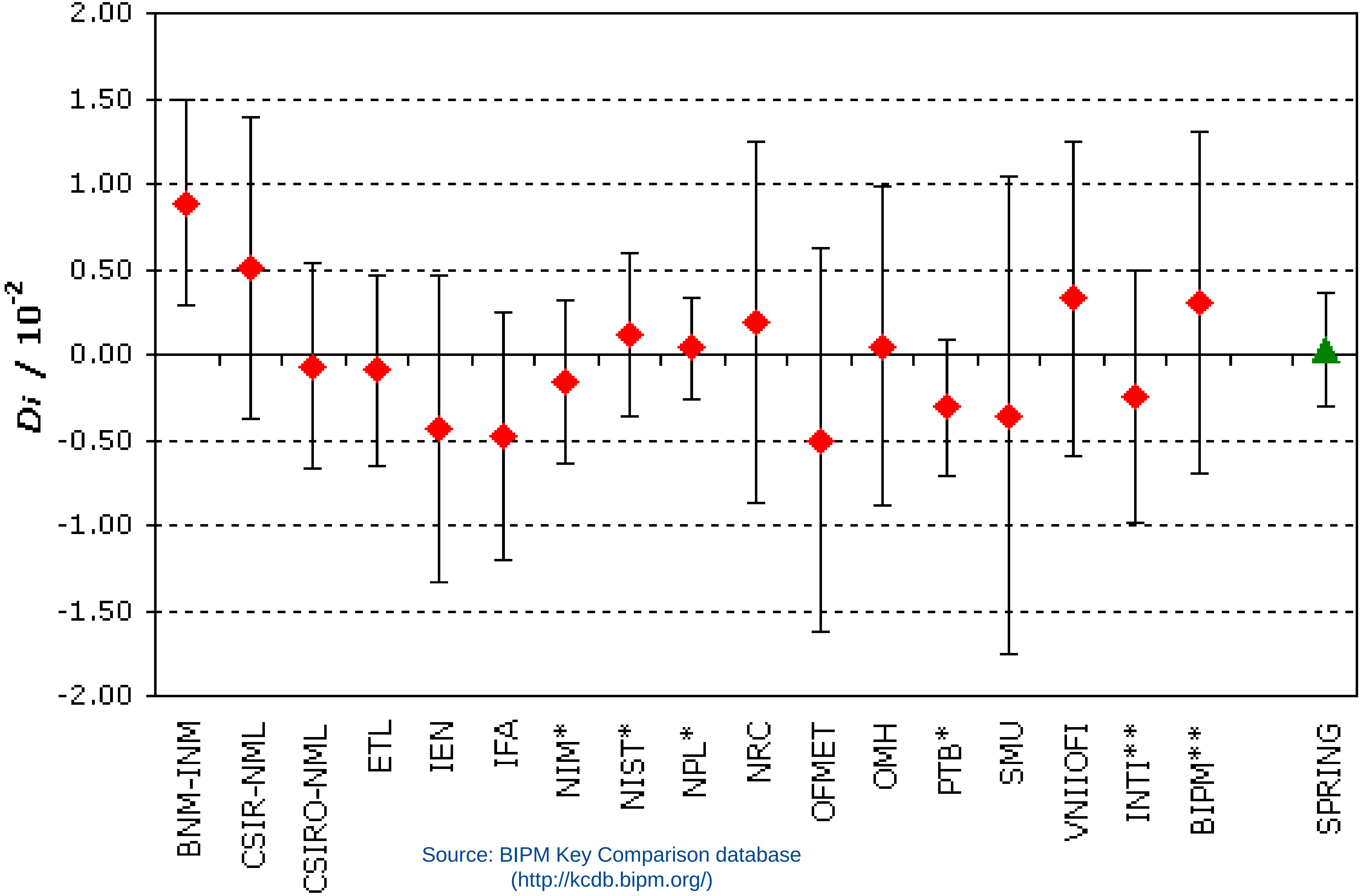
kg	kilogram	Mass
m	metre	Length
s	second	Time
A	ampere	Electric Current
K	kelvin	Temperature
cd	candela	Luminous Intensity
mol	mole	Amount of Substance

KC: Luminous intensity



Source: BIPM Key Comparison database
(http://kcdb.bipm.org/)

The GUM

JCGM 100:2008
GUM 1995 with minor corrections

Evaluation of measurement data — Guide to the expression of uncertainty in measurement

Évaluation des données de mesure — Guide pour l'expression de l'incertitude de mesure

First edition September 2008

© JCGM 2008

The Guide to the expression of Uncertainty in Measurement (GUM)

- The foremost authority and guide to the expression and calculation of uncertainty in measurement science
- Written by the JCGM and BIPM
- Covers a wide number of applications
- Technical with formal mathematics

http://www.bipm.org/en/publications/guides/gum.html

Uncertainty – Error – Correction

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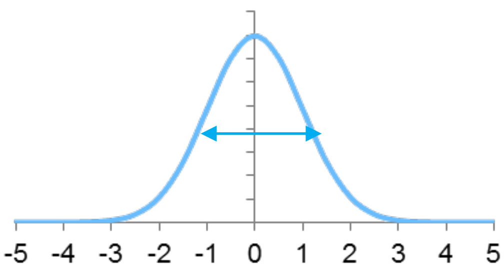
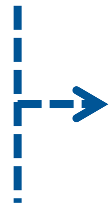
Uncertainty



Describes the spread

Drawn from a probability distribution described by uncertainty

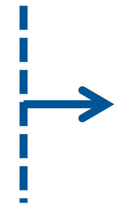
Error



Difference to the (unknowable) true value

Residual, uncorrectable, unknown error

Correction



Known offset from true value

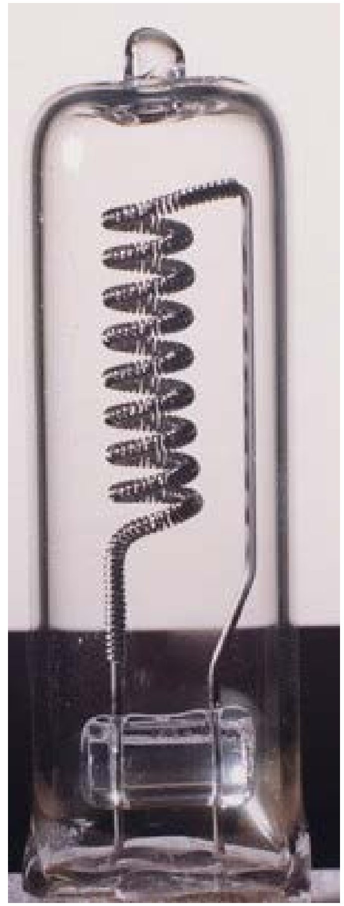
Traceability

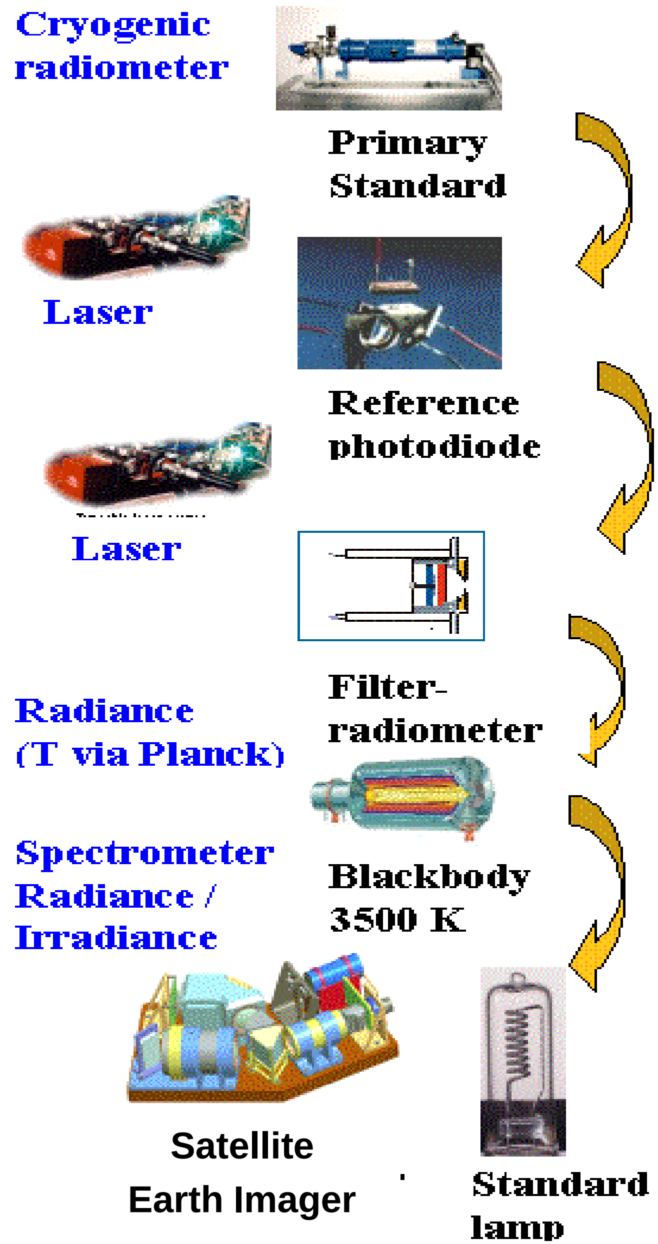
"Property of a measurement result relating the result to a stated metrological reference (free definition and not necessarily SI) through an unbroken chain of calibrations of a measuring system or comparisons, each contributing to the stated measurement uncertainty"

Committee on Earth Observation Satellites (CEOS)

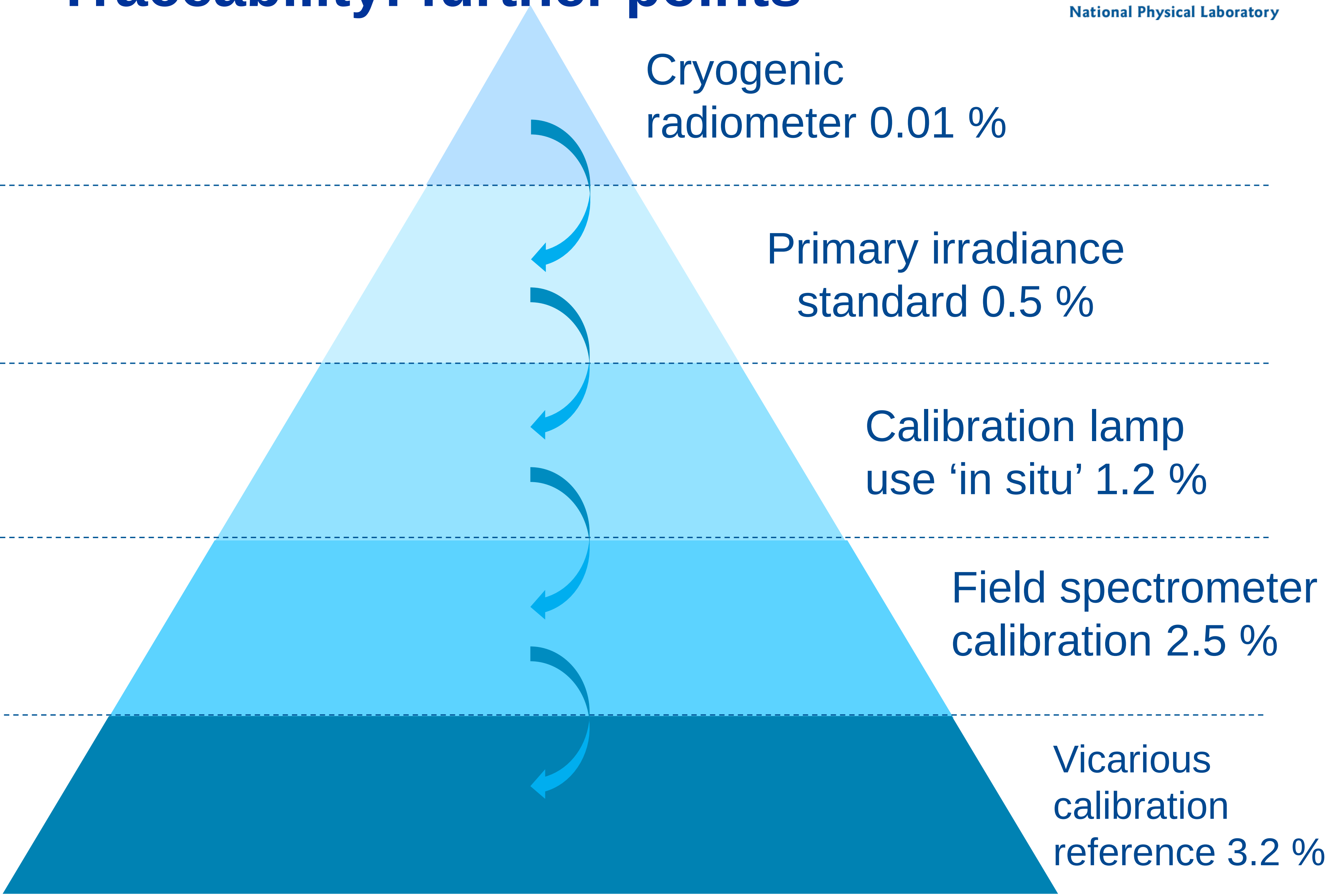
Spectral Radiance and Irradiance

SI Units



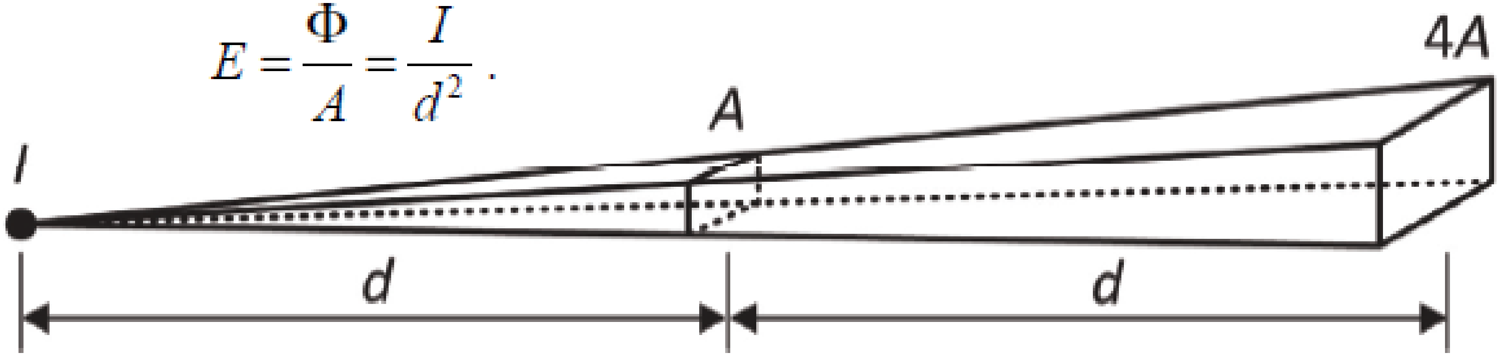


Traceability: further points



Inverse Square Law of Irradiance

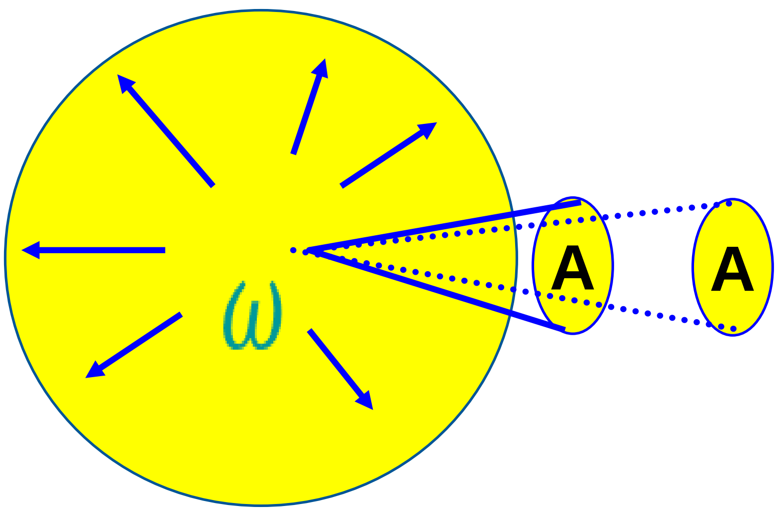
$$E = \frac{\Phi}{A} = \frac{I}{d^2}.$$



Palmer J.M, 2010

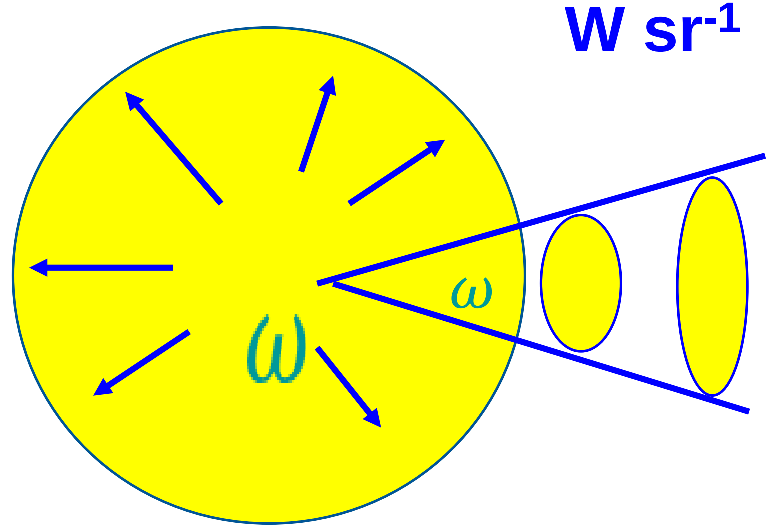
***E* Irradiance**

W m^{-2}



***I* Intensity**

W sr^{-1}



Irradiance Standards

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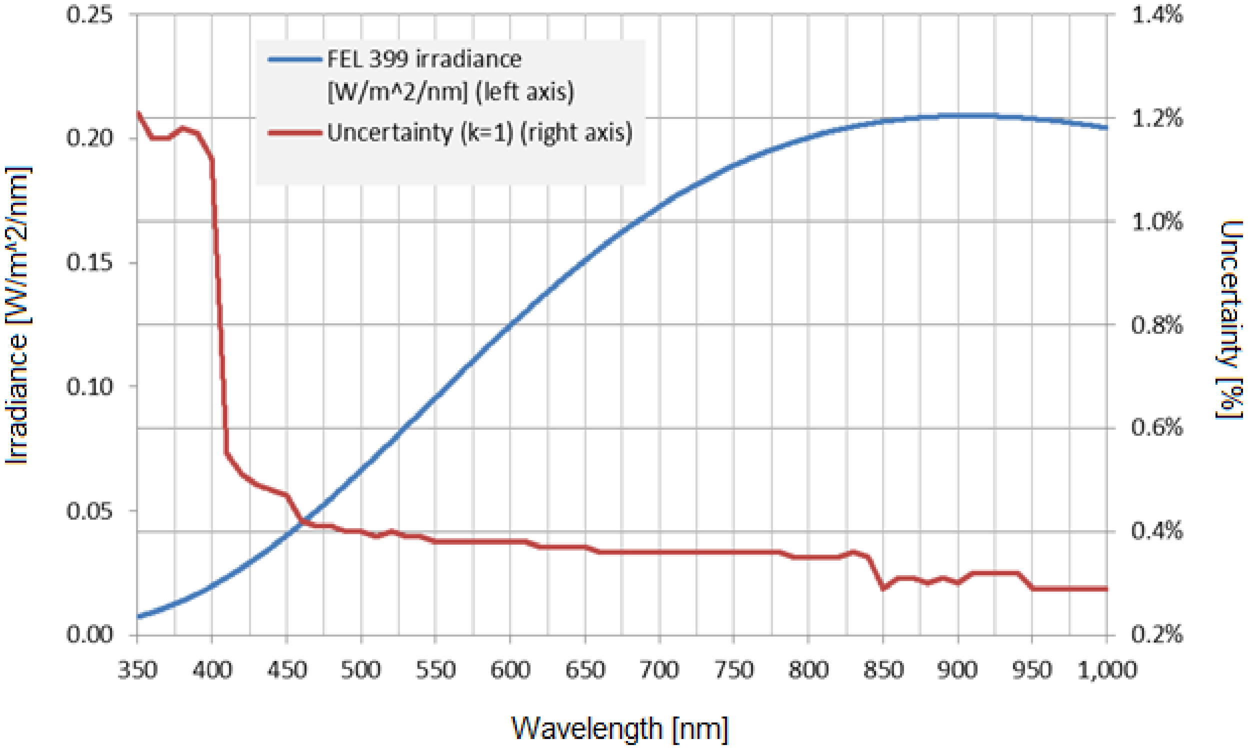
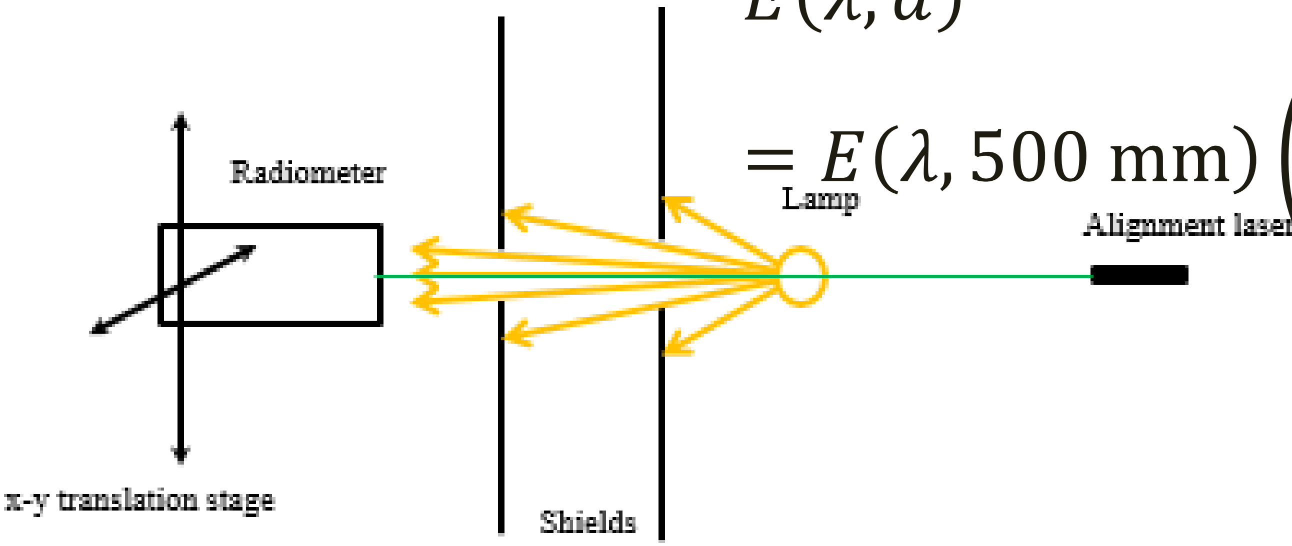
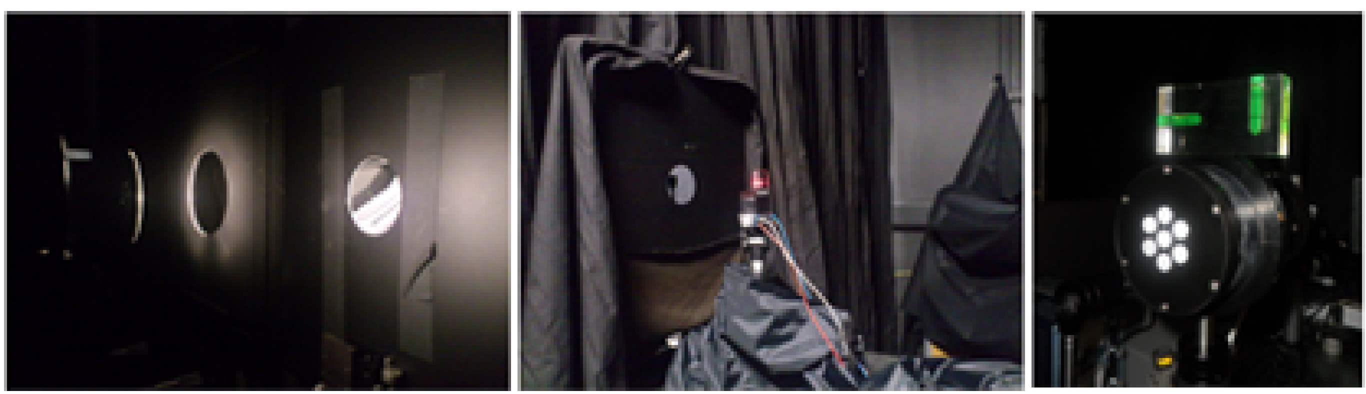
- Tungsten-halogen lamp (FEL) 1 kW (~ 3000 K)



Irradiance

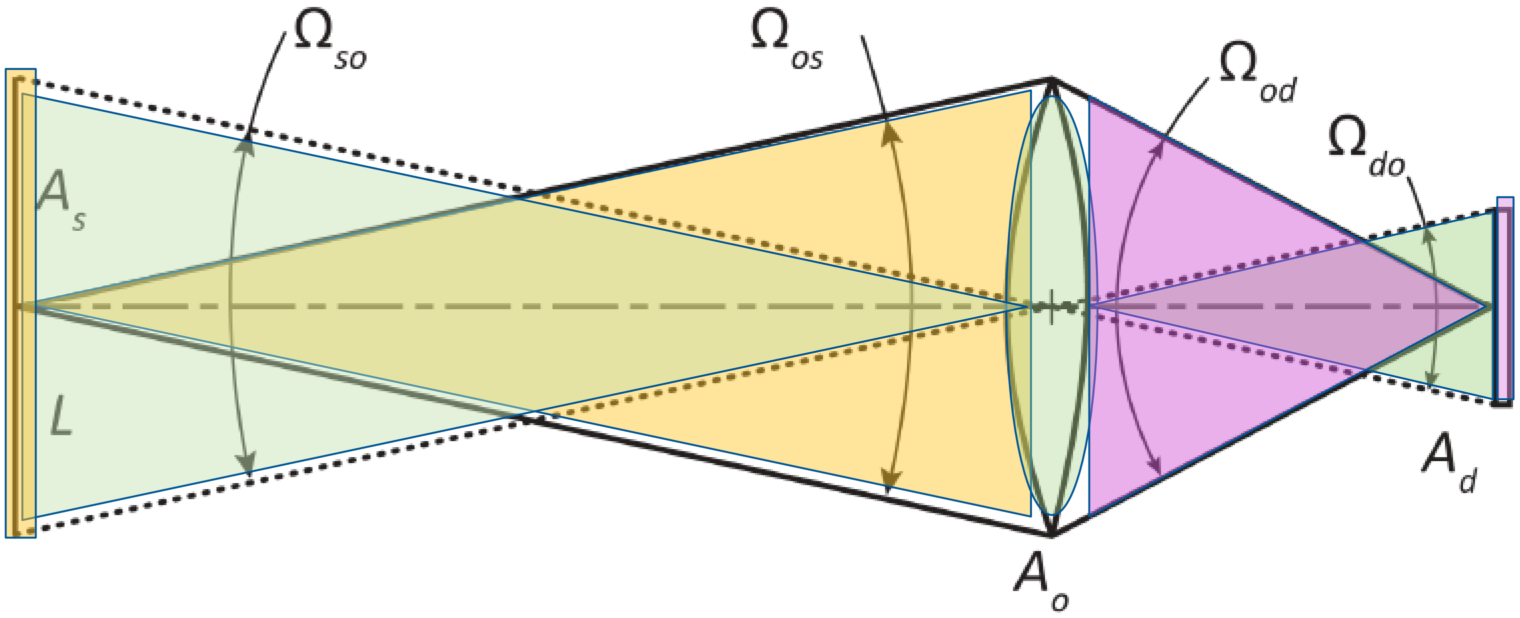
$$E(\lambda, d) = E(\lambda, 500\ \text{mm}) \left(\frac{500\ \text{mm}}{d} \right)^2$$





Radiance invariance

- Throughput invariance (étendue) $T = A\Omega$



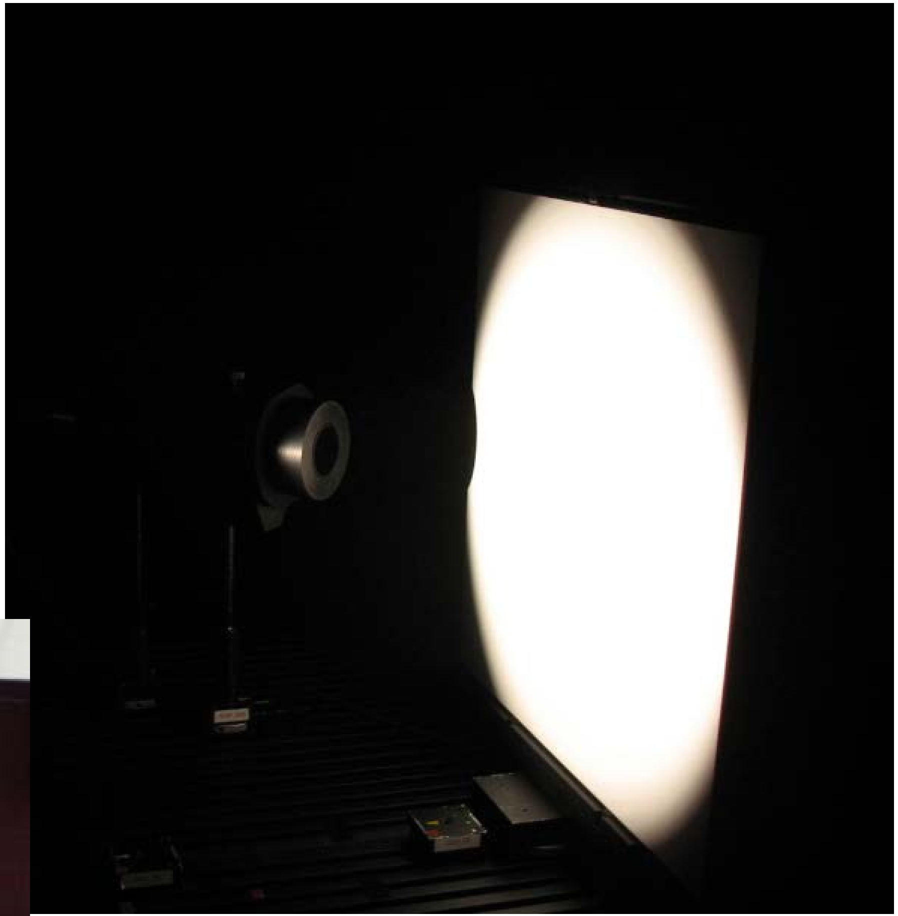
$$A_s\Omega_{os} = A_o\Omega_{so} = A_o\Omega_{do} = A_d\Omega_{od} \; .$$

Assuming lossless beam propagation and no lens transmission

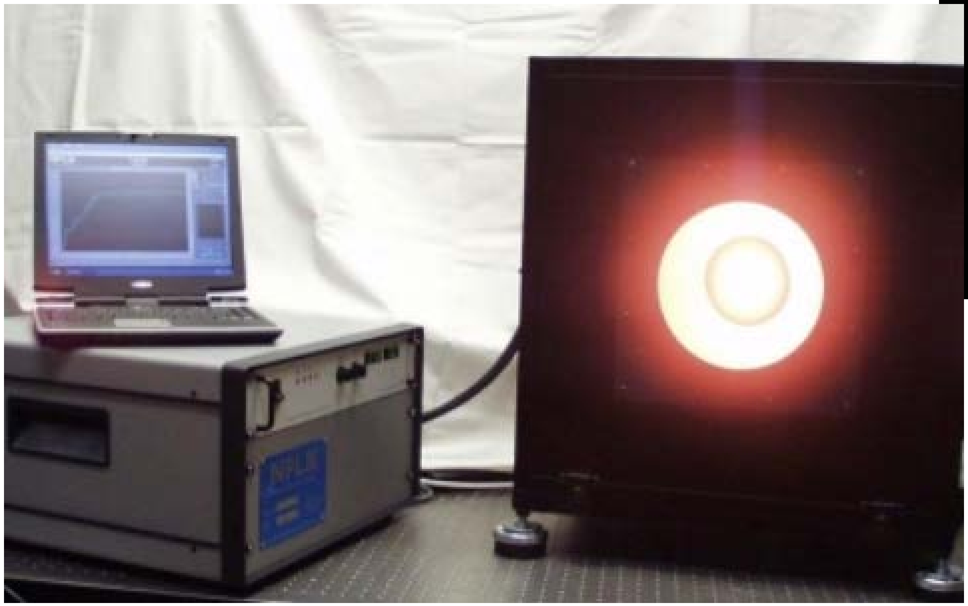
Palmer J.M, 2010

Radiance standards

- Lamp –reflectance standard

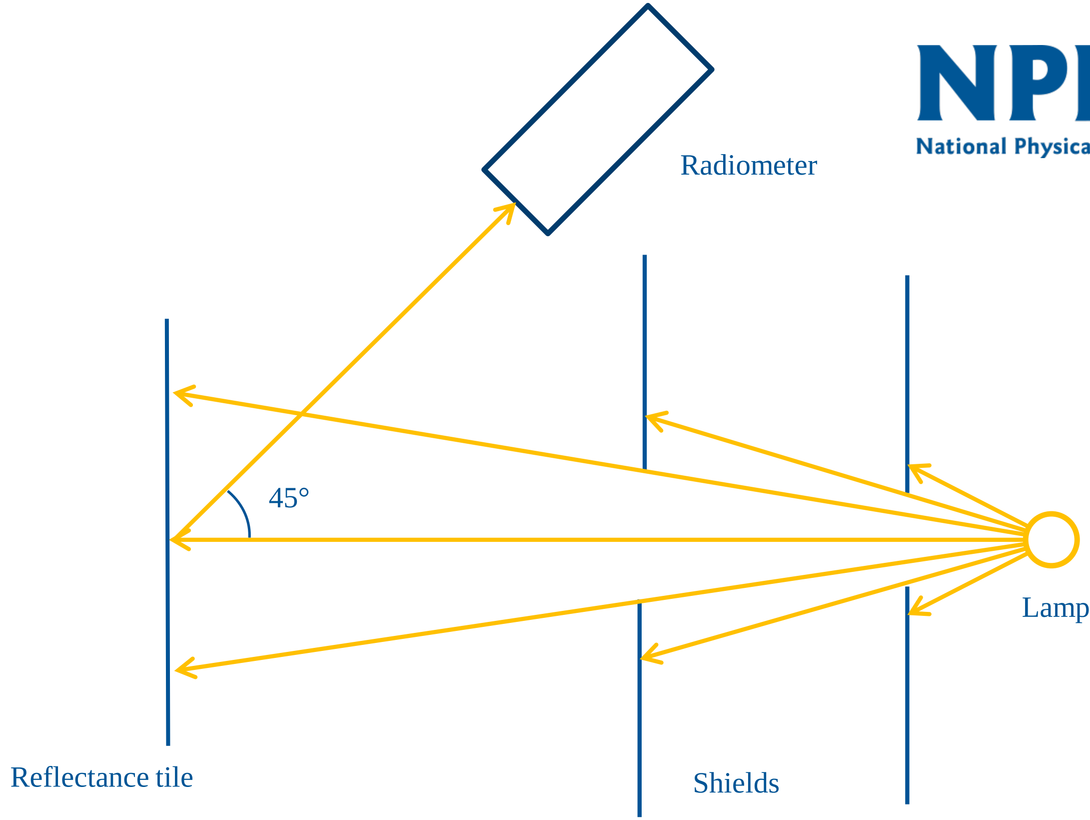


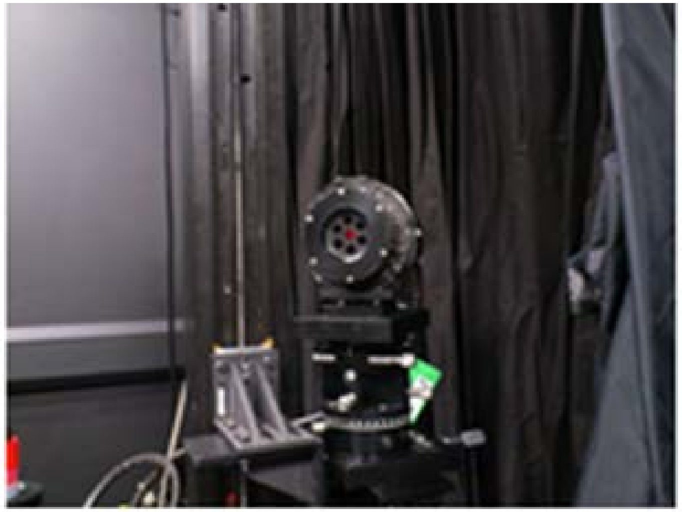
- Integrating sphere



Radiance

Lamp - tile





$$L_s = \frac{E_{FEL}\beta_{0:45}}{\pi}\frac{d_{cal}^2}{d_{use}^2}$$

Reflectance ρ and Reflectance factors

We have 9 kinds of reflectance and 9 equivalent reflectance factors.

First defined in 1977 by Nicodemous to simply surface scattering phenomena,

Assumptions:

Geometrical ray optics

A flat surface that is uniformly illuminated

Incident radiance depends only on direction

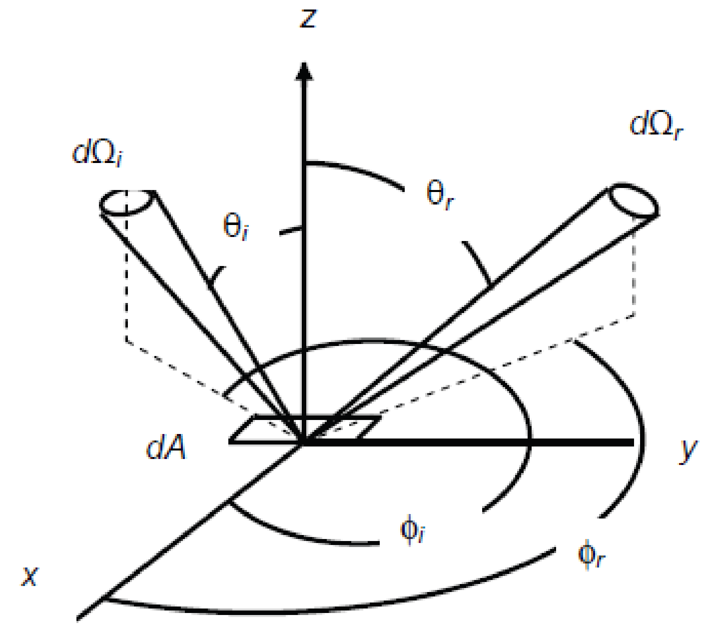
Surface has uniform and isotropic scattering properties

Nicodemous, 1977

BRDF bidirectional reflectance function

Definition

$$f(\theta_{\mathrm{I}},\varphi_{\mathrm{I}};\theta_{\mathrm{R}},\varphi_{\mathrm{R}},\lambda)=\frac{dL_{\mathrm{R}}(\theta_{\mathrm{I}},\varphi_{\mathrm{I}};\theta_{\mathrm{R}},\varphi_{\mathrm{R}},\lambda,E_I)}{dE_{\mathrm{I}}(\theta_{\mathrm{I}},\varphi_{\mathrm{I}})}$$



Measurement equation

BRDF=

$$f_r(\theta_{\mathrm{I}},\varphi_{\mathrm{I}},\theta_{\mathrm{R}},\varphi_{\mathrm{R}},\lambda)=d\rho(\theta_{\mathrm{I}},\varphi_{\mathrm{I}},\theta_{\mathrm{R}},\varphi_{\mathrm{R}},\lambda)=\lim_{\Omega\to 0}\frac{\Phi_{\mathrm{R}}(\theta_{\mathrm{I}},\varphi_{\mathrm{I}},\theta_{\mathrm{R}},\varphi_{\mathrm{R}},\lambda)}{\Phi_{\mathrm{I}}(\lambda)\cos\theta_{\mathrm{R}}\Omega}$$

BRF bidirectional reflectance factor

$$BRF\left(\theta_i\ ;\theta_r,\lambda\right) = BRDF \cdot \pi = \lim_{\Omega \to 0} \frac{\Phi_{\mathrm{R}}(\theta_{\mathrm{I}},\theta_{\mathrm{R}},\lambda)}{\Phi_{\mathrm{I}}(\lambda)\cos\theta_{\mathrm{R}}\Omega} \cdot \pi$$

the ratio of the radiance flux actually reflected by a sample surface to that which would be reflected into the same reflected-beam geometry by an ideal perfectly diffuse standard surface irradiated in exactly the same way as the sample

Radiometers

- Multispectral

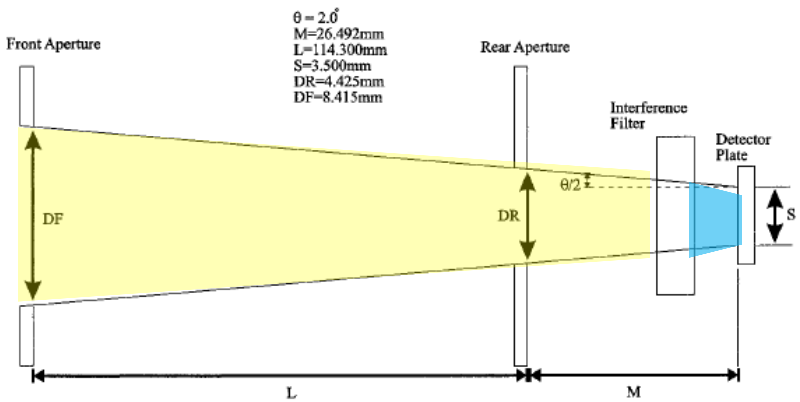
Stable and reliable

Limited spectral information

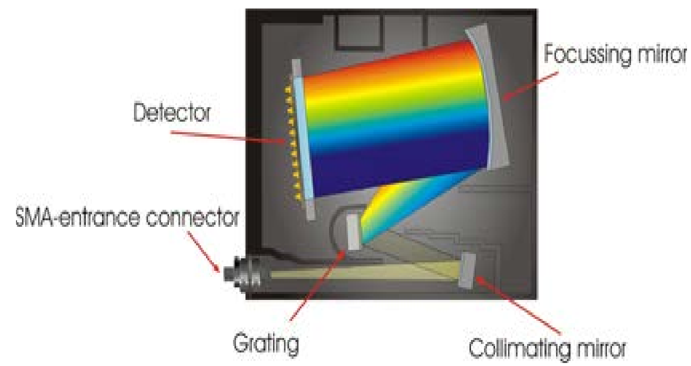
- Hyperspectral

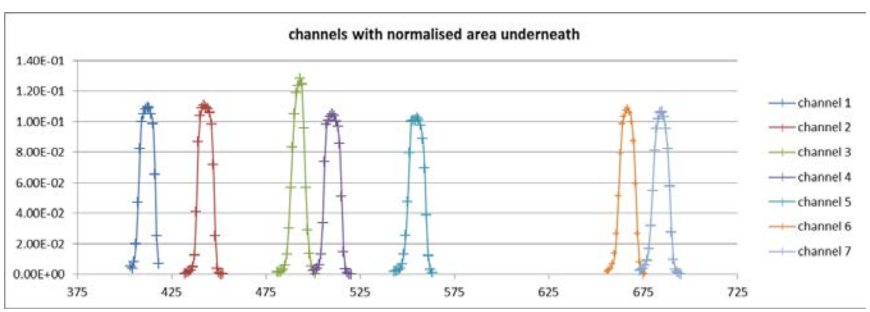
Demanding characterisation necessary !

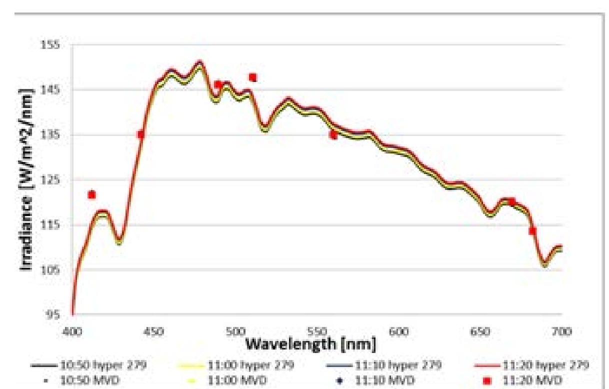
“Full” spectral information



Ehsani et al. 1998





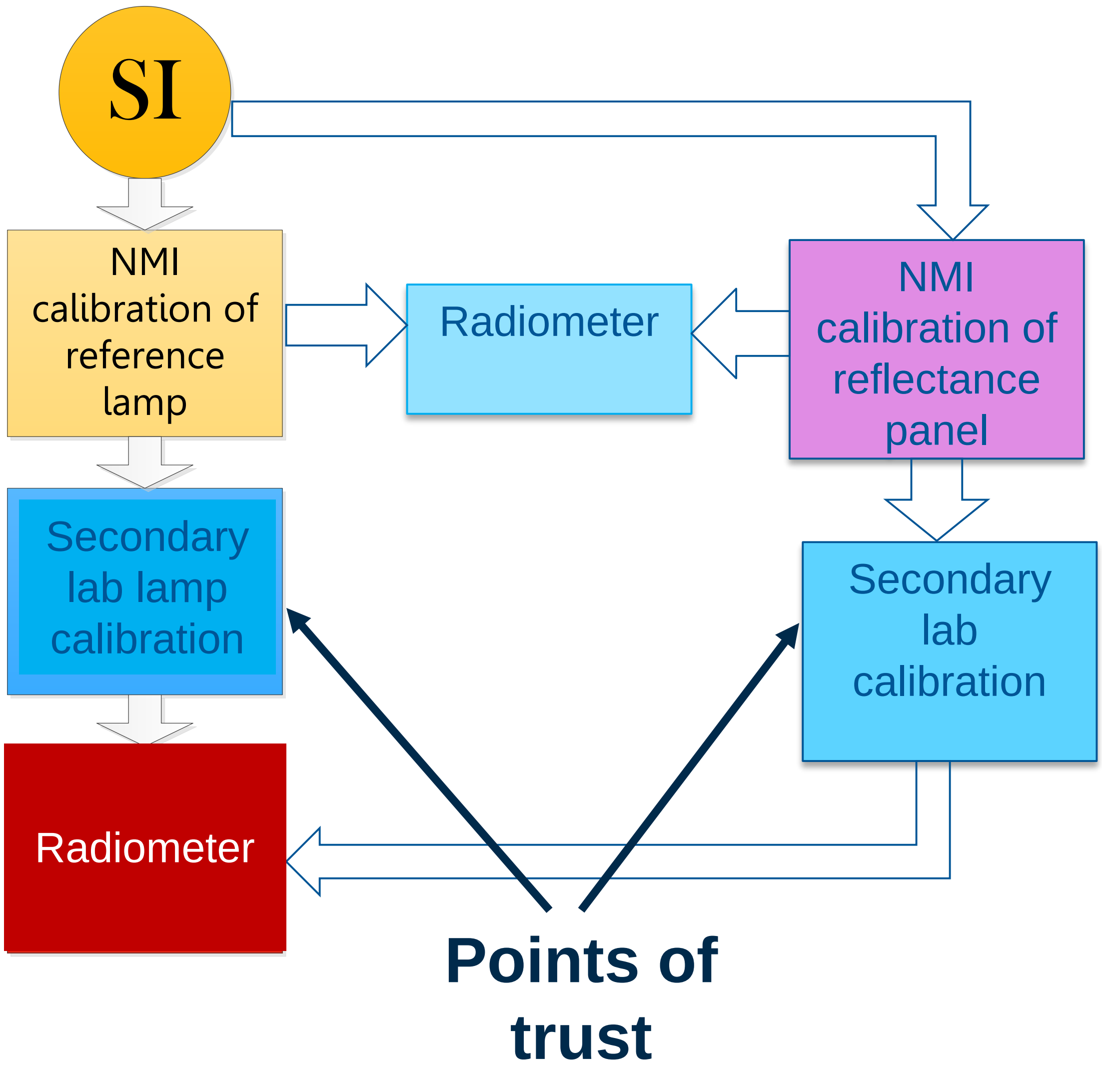


Steps to an uncertainty budget

1. Traceability Chain
2. Calculation Equation
3. Sources of Uncertainty
4. Measurement Equation
5. Sensitivity Coefficients
6. Assigning Uncertainties
7. Combining your uncertainties

Symbol	Uncertainty component	Size of effect	Correction applied?	Residual uncertainty	Divisor	Sensitivity coefficient	Uncertainty associated with final value due to effect
Combined standard uncertainty							
Expanded uncertainty							

Step 1: Traceability chain



Step 2: Calculation equations

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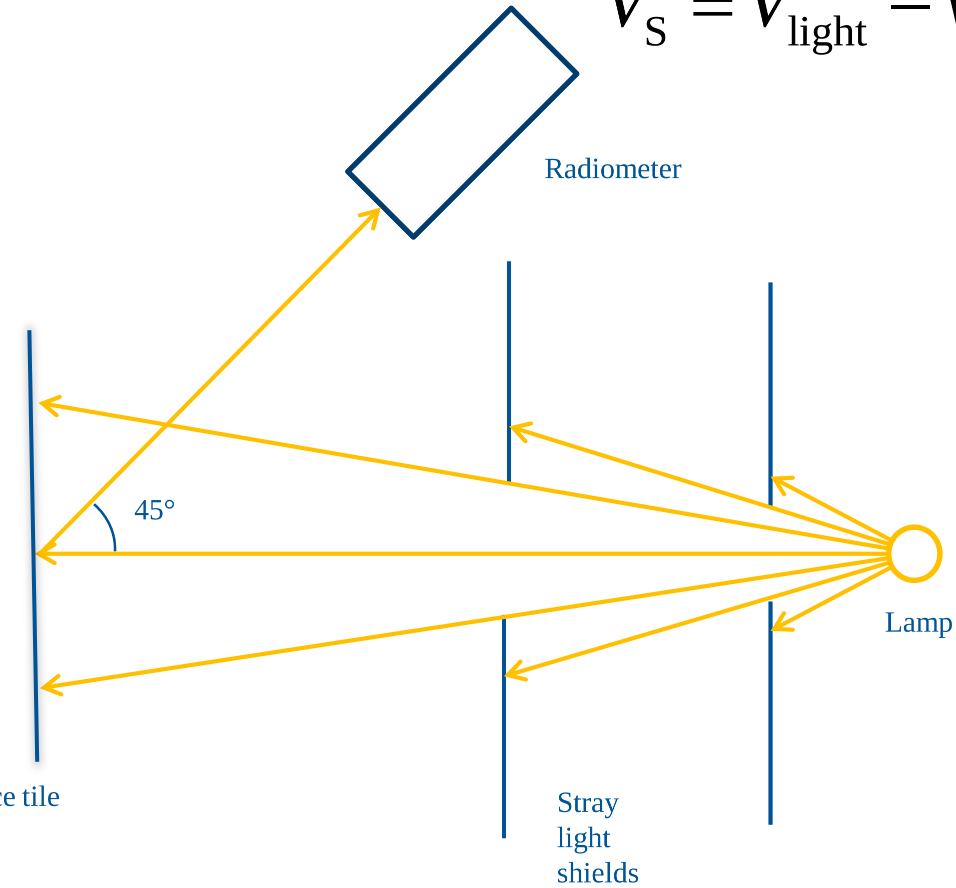
Absolute calibration coefficient

$$A_L = \frac{L_s}{V_s}$$

$$V_S = V_{\text{light}} - V_{\text{dark}}$$

$$L_s = \frac{E_{FEL}\beta_{0:45}}{\pi}\frac{d_{cal}^2}{d_{use}^2}$$



Step 3: Sources of uncertainty

Lamp

Lamp additional effects

- Ageing
- Alignment
- Current stability

Diffuser

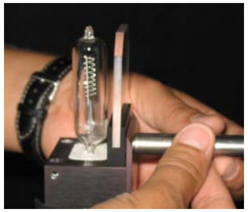
Diffuser additional effects

- Ageing
- Uniformity

Random noise

Instrument additional effects

- Stability (drift)
- Room stray light



Distance accuracy

Step 4: Measurement equations

$$L_s = \frac{E_{FEL}\beta_{0:45}}{\pi}\frac{d_{cal}^2}{d_{use}^2}$$

$$L_s = \frac{E_{FEL}\beta_{0:45}}{\pi}\frac{d_{cal}^2}{d_{use}^2} K_{lamp\_stab} K_{align} K_{current} K_{diff\_stab} K_{unif}$$

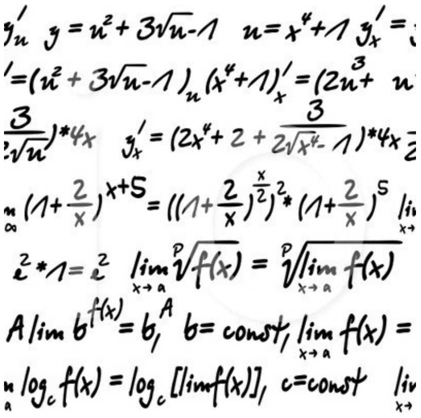
$$V_S = V_{light} - V_{dark}$$

$$V_S = V_{light} K_{light\_stab} + K_{stray} - V_{dark} K_{dark\_stab}$$

Determining the sensitivity coefficients

- Do an experiment

- Analytical expression



- Model it

Uncertainties of calculation equation components (1)

From calibration certificates

Component x_i	Sensitivity coefficient $c_i = \frac{\partial f}{\partial x_i}$	Relative radiance uncertainty due to … $c_i u(x_i)$
Lamp irradiance E_{FEL}	L_s / E_{FEL}	$1 \cdot u(E_{FEL}) / E_{FEL}$
Radiance factor $\beta_{0:45}$	$L_s / \beta_{0:45}$	$1 \cdot u(\beta_{0:45}) / \beta_{0:45}$
Distance d_{use}^2	$-2L_s / d_{use}$	$-2 \cdot u(d_{use}) / d_{use}$

$$\left(\frac{u(L)}{L}\right)^2 = \left(\frac{u(E_{FEL})}{E_{FEL}}\right)^2 + \left(\frac{u(\beta_{0:45})}{\beta_{0:45}}\right)^2 + (-2)^2 \left(\frac{u(d_{use})}{d_{use}}\right)^2$$

Certificate uncertainties

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$$L_s = \frac{E_{\mathrm{FEL}}\,\beta_{0:45}}{\pi}\frac{d_{\mathrm{cal}}^{2}}{d_{\mathrm{use}}^{2}}K_{\mathrm{lamp\_stab}}K_{\mathrm{align}}K_{\mathrm{current}}K_{\mathrm{diff\_stab}}K_{\mathrm{unif}}$$

From calibration certificates

Wave-length	Absolute Spectral Irradiance	Uncertainty
nm	mW m^{-2} nm^{-1}	%
545	4.37	1.5
550	4.54	1.5
555	4.72	1.5
590	5.96	1.4
595	6.14	1.4
600	6.31	1.3
605	6.48	
610	6.65	
615	6.83	
620	7.00	
625	7.18	
630	7.35	
635	7.52	
640	7.68	

Remember calibration certificates almost always quote uncertainties at *k* = 2 !

Certificate may not give required quantity (modelling may be needed to obtain desired quantity): read it carefully!

$$\rho \neq \beta_{0:45}$$

Integration and/or interpolation may need to be done if the wavelength values or radiometer bands do not match

Uncertainties of calculation equation components - sensitivity

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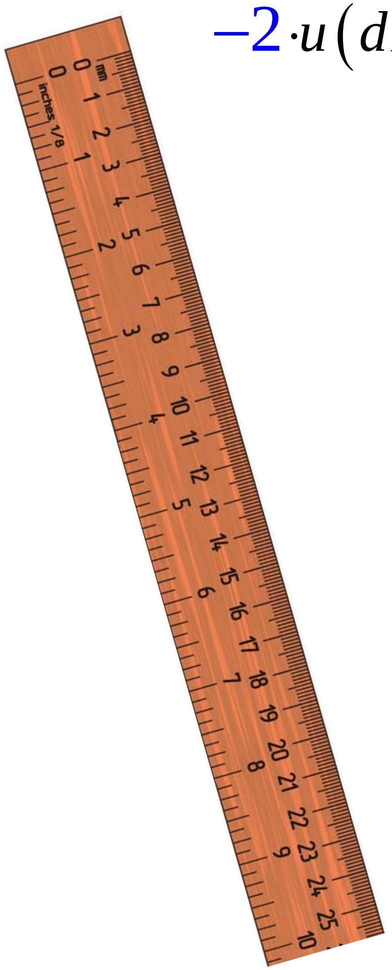
Component x_i	Sensitivity coefficient $c_i = \frac{\partial f}{\partial x_i}$	Relative radiance uncertainty due to … $c_i \mu(x_i)$
Lamp irradiance E_{FEL}	$L_\mathrm{s}/E_\mathrm{FEL}$	$1 \cdot u(E_\mathrm{FEL})/E_\mathrm{FEL}$
Radiance factor $\beta_{0:45}$	$L_\mathrm{s}/\beta_{0:45}$	$1 \cdot u(\beta_{0:45})/\beta_{0:45}$
Distance d_{use}^2	$-2L_\mathrm{s}/d_\mathrm{use}$	$-2 \cdot u(d_\mathrm{use})/d_\mathrm{use}$

$$\left(\frac{u(L)}{L}\right)^2 = \left(\frac{u(E_\mathrm{FEL})}{E_\mathrm{FEL}}\right)^2 + \left(\frac{u(\beta_{0:45})}{\beta_{0:45}}\right)^2 + (-2)^2\left(\frac{u(d_\mathrm{use})}{d_\mathrm{use}}\right)^2$$

Rectangular uncertainty distributions



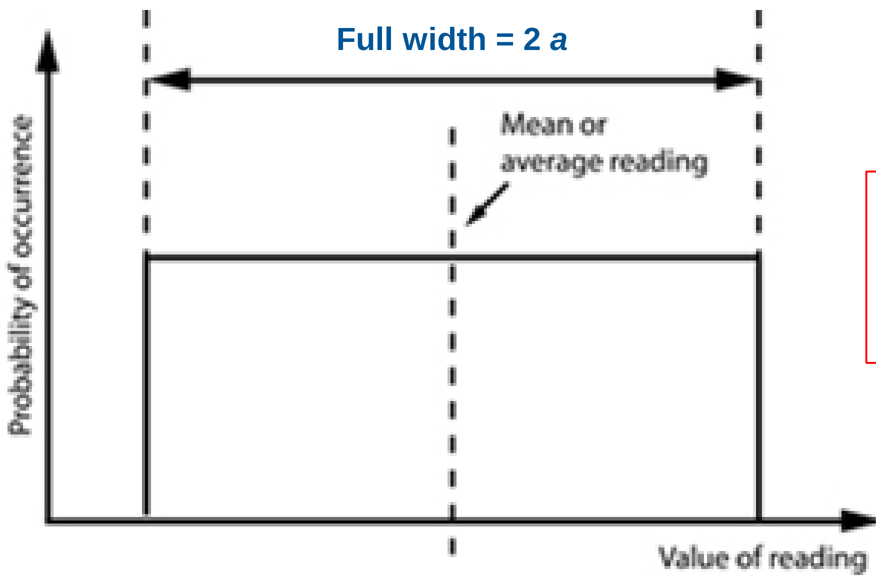
$$-2 \cdot u\left(d_{\text{use}}\right)/d_{\text{use}}$$

Resolution of distance measuring instrument = 0.1 mm

Measurement distance = 500.0 mm

Uncertainty associated with distance measurement =
(0.05 / 500) / √3 = 0.006 %

Uncertainty in irradiance from distance measurement =
2 × 0.006 % = 0.012 %



$$\boxed{\frac{a}{\sqrt{3}}}$$

Uncertainties of calculation equation components

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$$\left(\frac{u(L)}{L}\right)^2 = \left(\frac{u(E_{\mathrm{FEL}})}{E_{\mathrm{FEL}}}\right)^2 + \left(\frac{u(\beta_{0:45})}{\beta_{0:45}}\right)^2 + (-2)^2\left(\frac{u(d_{\mathrm{use}})}{d_{\mathrm{use}}}\right)^2$$

Symbol	Uncertainty component	Size of effect	Correction applied?	Residual uncertainty	Divisor	Sensitivity coefficient	Uncertainty associated with final value due to effect
$u(E_{\mathrm{FEL}})$	Ref. lamp irradiance	1.5 %	N	1.5 %	2	1	0.75 %
$u(\beta_{0:45})$	Tile radiance factor	2.0 %	N	2.0 %	2	1	1.00 %
$u(d_{\mathrm{use}})$	Lamp distance (500 mm)	0.05 mm	N	0.01 %	$\sqrt{3}$	2	0.012 %
$u(d_{\mathrm{use}})$							
$u(d_{\mathrm{use}})$							
$u(d_{\mathrm{use}})$							
$u(d_{\mathrm{use}})$							
$u(d_{\mathrm{use}})$							
$u(d_{\mathrm{use}})$							
$u(d_{\mathrm{use}})$							
$u(d_{\mathrm{use}})$							
Combined standard uncertainty							
Expanded uncertainty							

Uncertainty of additional effects (1)

$$L_s = \frac{E_{\mathrm{FEL}}\beta_{0:45}}{\pi}\frac{d_{\mathrm{cal}}^2}{d_{\mathrm{use}}^2} K_{\mathrm{lamp\_stab}} K_{\mathrm{align}} K_{\mathrm{current}} K_{\mathrm{diff\_stab}} K_{\mathrm{unif}}$$

Statistical analysis of repeated measurements

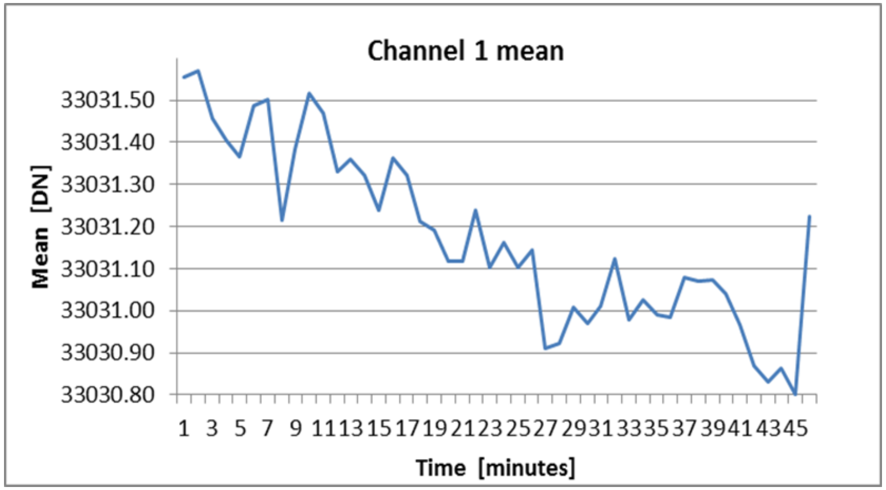
$$V_S = V_{\mathrm{light}} K_{\mathrm{light\_stab}} + K_{\mathrm{stray}} - V_{\mathrm{dark}} K_{\mathrm{dark\_stab}}$$

Repeat measurements with realignment of the lamp

Uncertainty of additional effects (2)

- Negligible instrument drift in controlled lab environment

0.7 DN change during
45 minute constant run



$$V_S = V_{\text{light}} \cancel{K_{\text{light\_stab}}} + K_{\text{stray}} - V_{\text{dark}} \cancel{K_{\text{dark\_stab}}}$$

Uncertainty of additional effects (3)

$$L_{\mathrm{s}} = \frac{E_{\mathrm{FEL}}\beta_{0:45}}{\pi}\frac{d_{\mathrm{cal}}^{2}}{d_{\mathrm{use}}^{2}} K_{\mathrm{lamp\_stab}} K_{\mathrm{align}} K_{\mathrm{current}} K_{\mathrm{diff\_stab}} K_{\mathrm{unif}}$$

Historical calibration records; data from other researchers

$$V_{\mathrm{S}} = V_{\mathrm{light}} K_{\mathrm{light\_stab}} + K_{\mathrm{stray}} - V_{\mathrm{dark}} K_{\mathrm{dark\_stab}}$$

Uncertainty of additional effects (4)

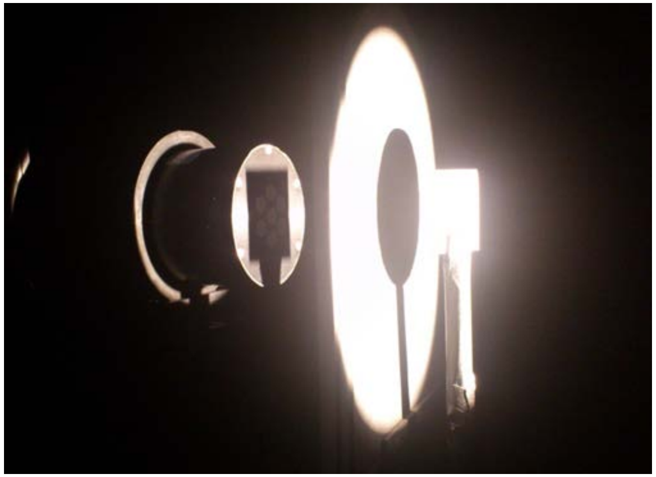
$$L_s = \frac{E_{\mathrm{FEL}}\beta_{0:45}}{\pi}\frac{d_{\mathrm{cal}}^2}{d_{\mathrm{use}}^2} K_{\mathrm{lamp\_stab}} K_{\mathrm{align}} K_{\mathrm{current}} K_{\mathrm{diff\_stab}} K_{\mathrm{unif}}$$

By measurement

$$V_S = V_{\mathrm{light}} K_{\mathrm{light\_stab}} + K_{\mathrm{stray}} - V_{\mathrm{dark}} K_{\mathrm{dark\_stab}}$$

Uncertainty of additional effects (5)

Difference between detector dark reading and measurement with detector FOV obscured smaller than standard deviation of individual dark runs



- Room stray light negligible

$$V_S = V_{light} K_{light\_stab} + \cancel{K_{stray}} - V_{dark} K_{dark\_stab}$$

Uncertainty of additional effects (6)

$$L_s = \frac{E_{\mathrm{FEL}}\beta_{0:45}}{\pi}\frac{d_{\mathrm{cal}}^2}{d_{\mathrm{use}}^2} K_{\mathrm{lamp\_stab}} K_{\mathrm{align}} K_{\mathrm{current}} K_{\mathrm{diff\_stab}} K_{\mathrm{unif}}$$

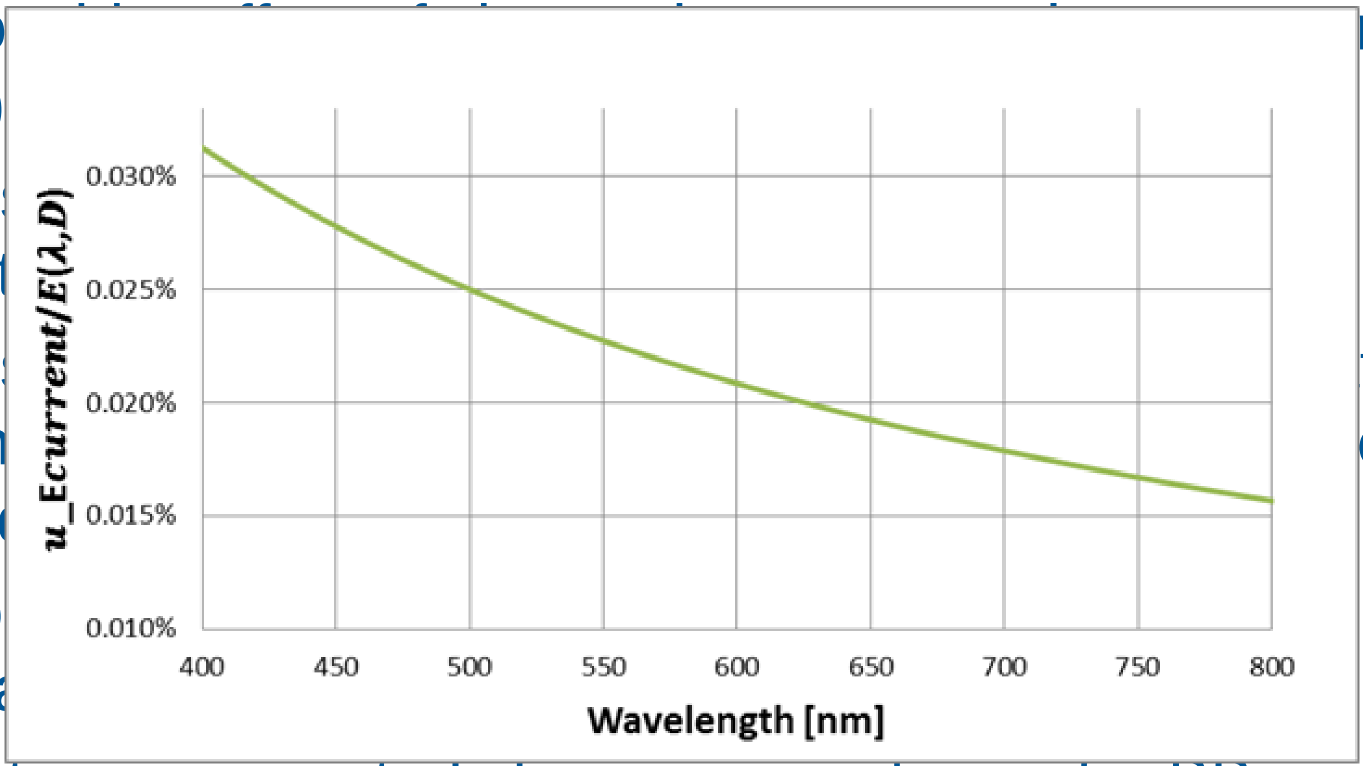
By modelling (or by measurement)

$$V_S = V_{\mathrm{light}} K_{\mathrm{light\_stab}} + K_{\mathrm{stray}} - V_{\mathrm{dark}} K_{\mathrm{dark\_stab}}$$

Uncertainty of additional effects (7)

Relationship between lamp current and irradiance **modelled** by:

1. Co… r ($P_{elec} \propto I^2$)
2. As… and opt…
3. As… tor for sm… ower an…
4. So… ative ch…



5. Determine spectral change in irradiance for BB associated with change in T

Treat current uncertainty as rectangular distribution

Lamp current effect by measurement

- Suppose:

  Lamp current uncertainty is 0.020 A

  Lamp current is 8.000 A

  Lamp CCT is 3000 K

- We can **measure** the effect by:

  Measuring irradiance at a current of 8.000 A

  Measuring irradiance at a current of (say) 7.800 A (needs to be large enough difference to enable effect to be measured reliably)

  $u(E_{\text{current}})$ = 0.1 x observed relative change in irradiance due to 0.200 A change in current

Uncertainty of additional effects (8)

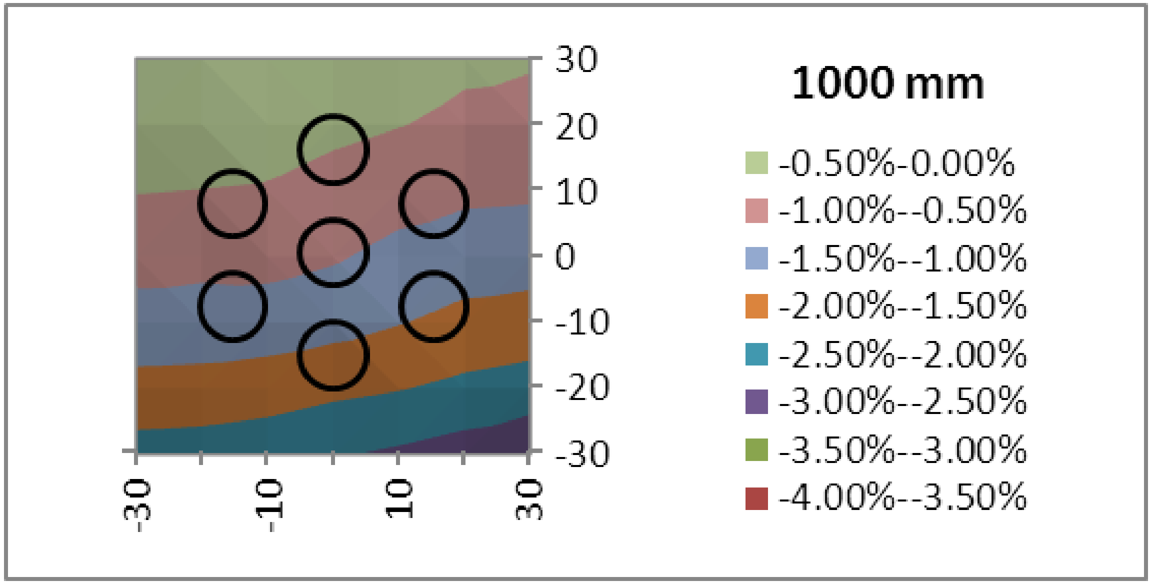
$$L_s = \frac{E_{\mathrm{FEL}}\beta_{0:45}}{\pi}\frac{d_{\mathrm{cal}}^2}{d_{\mathrm{use}}^2} K_{\mathrm{lamp\_stab}} K_{\mathrm{align}} K_{\mathrm{current}} K_{\mathrm{diff\_stab}} K_{\mathrm{unif}}$$

By combination of measurement and ‘modelling’

$$V_S = V_{\mathrm{light}} K_{\mathrm{light\_stab}} + K_{\mathrm{stray}} - V_{\mathrm{dark}} K_{\mathrm{dark\_stab}}$$

Uncertainty of additional effects (9)

Measured uniformity and consideration of where radiometer is placed



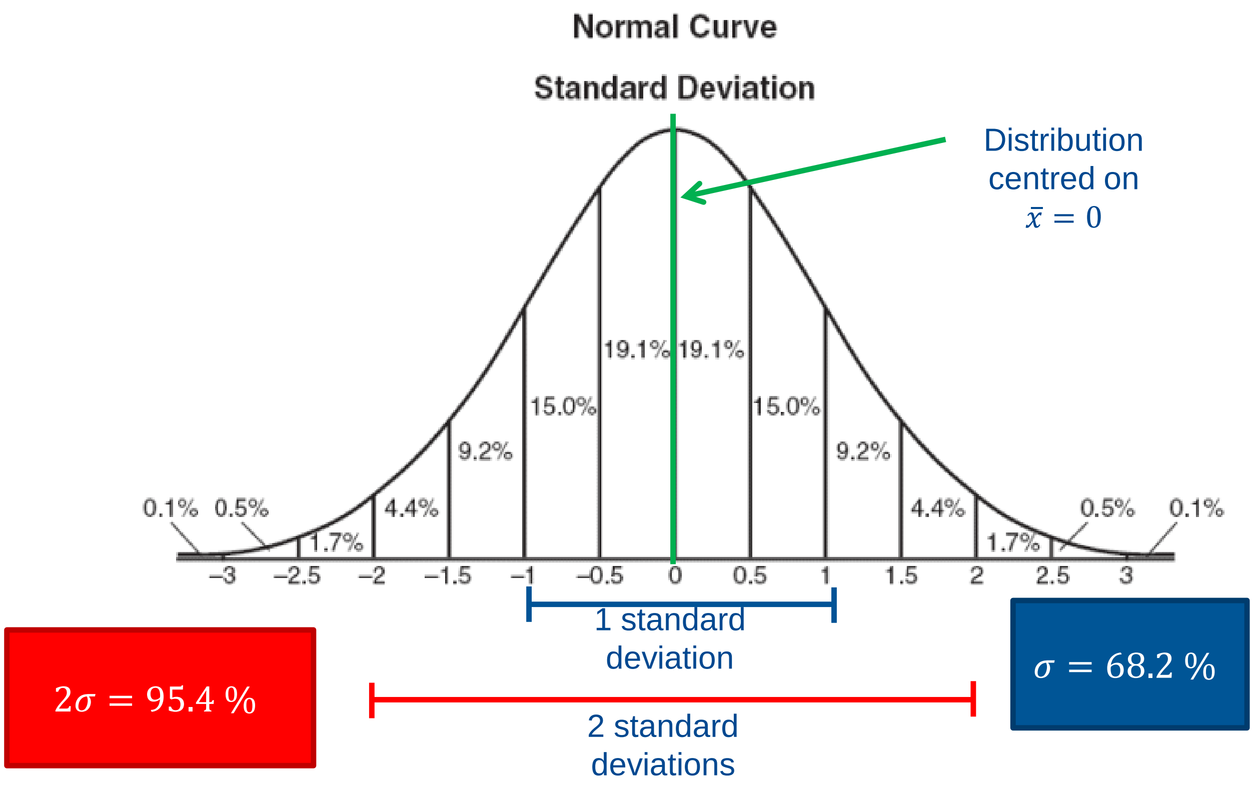
Worst case uncertainty is ± 1.5 %

Even if diffuser is perfectly uniform, non-uniformity due to lamp irradiance can be large, especially at short distances (inverse square law)

Combined uncertainty

Symbol	Uncertainty component	Size of effect	Correction applied?	Residual uncertainty	Divisor	Sensitivity coefficient	Uncertainty associated with final value due to effect
$u(E_{\mathrm{FEL}})$	Ref. lamp irradiance	1.5 %	N	1.5 %	2	1	0.75 %
$u(\beta_{0:45})$	Tile radiance factor	2.0 %	N	2.0 %	2	1	1.00 %
$u(d_{\mathrm{use}})$	Lamp distance (500 mm)	0.05 mm	N	0.01 %	√3	2	0.012 %
$u(K_{\mathrm{align}})$	Lamp alignment	0.15 %	N	0.15 %	1	1	0.15 %
$u(K_{\mathrm{l\_stab}})$	Light reading stability	negligible	N	negligible			negligible
$u(K_{\mathrm{d\_stab}})$	Dark reading stability	negligible	N	negligible			negligible
$u(K_{\mathrm{lamp\_stab}})$	Lamp stability	0.083 %	N	0.083 %	√3	1	0.048 %
$u(K_{\mathrm{diff\_stab}})$	Diffuser stability	0.125 %	N	0.125 %	√3	1	0.072 %
$u(K_{\mathrm{stray}})$	Stray light in lab	negligible	N	negligible			negligible
$u(K_{\mathrm{current}})$	Lamp current (8.000 A)	0.004 A	N	0.25 % in I, or 0.99 % in E_{FEL} at 600 nm	√3	1	0.572 % (at 600 nm)
$u(K_{\mathrm{unif}})$	Radiance uniformity	1.50 %	N	1.50 %	√3	1	0.866 %
Combined standard uncertainty							1.63 %
Expanded uncertainty (*k*=2)							3.3 %

Expanding uncertainties



If the distribution is not Gaussian, then a different coverage factor is needed.

Conclusions

1. Traceability Chain

    Show linkage back to ‘point of trust’

2. Calculation Equation

    Equation for each step in measurement process / step in chain

3. Sources of Uncertainty

    Consider all factors that may affect result for each calculation equation

4. Measurement Equation

    Include all sources of uncertainty

    Types of uncertainty (multiplicative, additive)

Conclusions

5. Sensitivity Coefficients

   Mathematically, from experimental investigations, or by modelling

6. Assigning Uncertainties

   Other information (e.g. certificates, historical data, other researchers), statistical analysis, experimental studies, modelling, or combination

7. Combining your uncertainties

   When to stop

8. Expanding your uncertainties

Summary

Metrology view

- SI traceability – ensures the valid measurements
- Calibration links instrument output readings to physical values
- SI traceability especially important to radiometry, as these measurements are used calibration and validation of satellite sensors
- Instrument characteristics influence their properties and performance

Thank you

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